SECTION 5

Hydraulics - General

Introduction: The development of this section is based on the assumption that users will have available, as a working tool, a copy of the "Handbook of Hydraulics", Third Edition, by Horace W. King, McGraw-Hill Book Company. Those engineers whose work includes an appreciable amount of hydraulic computations will find time-saving, tabulated material in "Hydraulic Tables" by the War Department, Corps of Engineers, U. S. Government Printing Office. For brevity these two books are subsequently referred to as "King's Handbook" and "Hydraulic Tables."

A partial list of other widely used publications dealing with the practical phases of hydraulics and hydraulic structures is given below. The need for and the usefulness of these or other handbooks not listed will depend on the amount and type of work encountered in different work unit areas. Inclusion in this list is not a recommendation for the books listed nor a recommendation against any book not listed.

Handbook of Water Control - published by the R. Hardesty Mfg. Co.
Handbook of Culvert and Drainage Practice - published by Armco Culvert Mfg. Assn.
Handbook of Welded Steel Pipe - published by Armco Drainage and Metal Products, Inc., successors to R. Hardesty Mfg. Co.
Concrete Pipe Lines - published by American Concrete Pipe Assn.
Low Dams - by a Subcommittee of the National Resources Committee, U. S. Government Printing Office.
Hydraulic and Excavation Tables - Bureau of Reclamation, Department of Interior, U. S. Government Printing Office.

1. Symbols and Units

1.1 Symbols. The symbols used and their definitions are:

a = cross-sectional area.

b = bottom width of channel.

C = coefficient of discharge for weirs and orifices; constant in Hazen-Williams formula.

D = diameter of circular section.

d = depth of flow normal to channel bottom; diameter of pipe in feet.

d_a = average depth of flow in a channel reach.

d_c = critical depth of flow perpendicular to channel bottom.

d_i = diameter of pipe in inches.

d_m = mean depth of flow at a section.

d_n = depth of normal flow; that is, depth of uniform flow.

F = force.

g = acceleration of gravity.

H = total head.

H_e = specific energy head.

h_f = friction head.

h_p = pressure head.

h_v = velocity head.

I = volume of inflow to a reservoir.

i = rate of inflow to a reservoir.

K and K' = factors used in certain arrangements of Manning's formula and which vary with the ratios of specified linear dimensions of cross sections.

K = head loss coefficient. In most cases this symbol is used with a subscript to make it specific and where so used it is clearly defined.

L = length of channel or closed conduit; length of rectangular weir crest.

ℓ = length of a portion of a channel or closed conduit.

M = mass

n = coefficient of roughness in Manning's formula; an exponent.

O = volume of outflow from a reservoir.

o = rate of outflow from a reservoir.

P = total pressure force; a symbol used in a certain arrangement of Manning's formula, the value of which is $\dfrac{n^2}{2.2082\ ar^{4/3}}$

P_H = horizontal component of pressure force.

P_R = resultant pressure force.

P_V = vertical component of pressure force.

p = intensity of pressure per unit of area; wetted perimeter.

Q = total discharge; that is, volume of flow per unit of time.

Q_c = critical discharge.

Q_n = normal discharge.

q = discharge per unit of width.

q_c = critical discharge per unit of width.

q_n = normal discharge per unit of width.

R = Reynold's number

$R = \dfrac{s_o}{2}\dfrac{d\ell}{dd}$

r = hydraulic radius.

r_m = mean hydraulic radius in channel reach.

S = volume of temporary reservoir storage.

s = slope; that is, the tangent of the angle a line makes with the horizontal; the slope of the energy gradient in Manning's formula.

s_c = critical slope.

s_f = friction slope.

s_o = slope of channel bottom.

T = width of flow at the water surface; a conversion-time interval.

t = time.

V = volume.

v = mean velocity of flow.

v_a = velocity of approach.

v_c = critical velocity.

v_n = normal velocity; that is, velocity of uniform flow.

W = weight.

w = unit weight.

x = a horizontal distance or abscissa; an exponent; a variable; a time-conversion factor.

y = a vertical distance or ordinate; a variable.

$\bar{x}$, $\bar{y}$ = coordinates of the center of gravity of an area.

z = the elevation of a specified point above datum; the slope of the sides of trapezoidal sections expressed as a ratio of horizontal to vertical.

α (Greek alpha) = a kinetic energy correction factor.

β (Greek beta) = an angle defined specifically where used.

θ (Greek theta) = an angle defined specifically where used.

ν (Greek nu) = kinematic viscosity.

1.2 Units of the foot-pound-second system are used unless others are specified. Factors to be used in making conversions between various units and dimensions are available in Tables 1 to 11, "King's Handbook."

In many cases the conversion of units and dimensions is looked upon as a simple, unimportant process. The fact is that conversions are a frequent source of error in engineering computations. Valid equations must be expressed in corresponding units; that is, in a true equation there must be equality between both units and numbers. Engineers can materially reduce the chance of conversion errors by forming the habit of thinking in terms of equality of units as well as equality of numbers.

Problems often arise in which the correct relationship between units and dimensions is not readily visualized and becomes clear only by analysis. In these situations the quick selection of one or a series of conversion factors not expressed in equation form and tested for validity, may result in costly, systematic errors. As examples of the use of sound principles in the conversion process, consider the following:

Example 1:

1 cubic meter = Y gallons

Basically this intends to express an equality between two volumes; that is, two different linear dimensions raised to the third power. Since cubic meters can no more be equated to gallons than freight cars can be equated to bicycles, it is evident that some factor having dimensional as well as numerical value must be introduced if the expression is to be made a valid equation. Analysis shows that:

$$1 \text{ m}^3 \times \frac{35.3145}{1} \frac{\text{ft.}^3}{\text{m}^3} \times \frac{1728}{1} \frac{\text{in.}^3}{\text{ft.}^3} \times \frac{1}{231} \frac{\text{gal.}}{\text{in.}^3} = 264.17 \text{ gal.}$$

Note that all dimensions on the left cancel, leaving the unit, gallon; that is, corresponding units, on each side of the equation. The analysis results in a general equation for conversions between cubic meters and gallons:

$$\text{X m}^3 \times 264.17 \frac{\text{gal.}}{\text{m}^3} = \text{Y gal.}$$

Example 2:

1 acre-foot per hour = Y gallons per minute

Step by step analysis results in a valid conversion equation consistent in both units and dimensions:

$$1 \frac{\text{ac.-ft.}}{\text{hr.}} \times \frac{43560}{1} \frac{\text{ft.}^2}{\text{ac.}} \times \frac{1}{60} \frac{\text{hr.}}{\text{min.}} \times \frac{7.4805}{1} \frac{\text{gal.}}{\text{ft.}^3} = 5431 \frac{\text{gal.}}{\text{min.}}$$

or

$$\text{X} \frac{\text{ac.-ft.}}{\text{hr.}} \times 5431 \frac{\frac{\text{gal.}}{\text{min.}}}{\frac{\text{ac.-ft.}}{\text{hr.}}} = \text{Y} \frac{\text{gal.}}{\text{min.}}$$

Example 3:

1 cubic foot per second-day = Y acre feet

Analysis results in:

$$1 \frac{\text{ft.}^3\text{-day}}{\text{sec.}} \times \frac{1}{43560} \frac{\text{ac.}}{\text{ft.}^2} \times \frac{24 \times 3600}{1} \frac{\text{sec.}}{\text{day}} = 1.9835 \text{ ac.ft.}$$

or

$$\text{X cfs.-d} \times 1.9835 \frac{\text{ac.ft.}}{\text{cfs.-d}} = \text{Y ac.ft.}$$

Engineers who will approach conversion problems by the use of the principles illustrated above should secure the following benefits: (1) freedom from conversion errors; (2) savings in time required for both original and "check" computations; and (3) accuracy of conversion factor selection from standard tables or other sources.

2. Hydrostatics

2.1 Unit hydrostatic pressure varies directly with the depth and the unit weight of water and is expressed by the equation:

$$p = wh \qquad (5.2\text{-}1)$$

p = intensity of pressure per unit of area.
w = unit weight of water.
h = depth of submergence, or head.

Useful working equations are:

p, in p.s.i. = 0.433 h, in ft.
p, in lb./ft.^2 = 62.4 h, in ft.

In a body of water with free surface, the total unit pressure is the sum of the liquid pressure and the atmospheric pressure. The majority of hydraulic structures are built and operate under conditions such that atmospheric pressures are balancing forces which may be neglected. However, when significant, atmospheric pressure should be fully considered and its effect upon hydraulic operation and structural stability determined. Examples of structures whose operation or stability may be affected by atmospheric pressure are pipe lines having a portion of their length above the hydraulic grade line; weirs with nonadhering nappe which do not have the under side of the nappe free to the atmosphere.

2.2 Pressure loadings. The analysis of structures under pressure loads will, in most cases, be facilitated by the use of pressure diagrams. Since unit pressure varies directly with head, diagrams showing the variation of unit pressure in any plane take the form of triangles, trapezoids, or rectangles. Typical pressure diagrams and aides to working with such diagrams are shown on drawing ES-31.

2.3 Buoyancy. A submerged body is acted on by a vertical, buoyant force equal to the weight of the displaced water.

$$F_B = Vw \qquad (5.2\text{-}2)$$

F_B = buoyant force.

V = volume of the body.

w = unit weight of water.

If the unit weight of the body is greater than that of water, there is an unbalanced, downward force equal to the difference between the weight of the body and of an equal volume of water, and the body will sink. If the body has a unit weight less than that of water, the body will float with part of its volume below and part above the water surface in a position such that:

$$W = Vw \qquad (5.2\text{-}3)$$

W = weight of the body.
V = volume of the body below the water surface, i.e. the volume of the displaced water.
w = unit weight of water.

Close examination should be made of the stability of hydraulic structures as it will be affected by: (1) whether the structure will be submerged; (2) whether wide variations in buoyant forces and net or effective weights are possible.

Porous materials, when submerged, are subject to different net weights and are acted on by different buoyant forces depending upon whether the voids are filled with air or water. Note the wide variation in the possible net weight of one cubic foot of treated structural timber weighing 55 lbs. under average atmospheric moisture conditions and having 30 percent voids:

1 ft.^{3} of structural timber, 30 percent voids	Before Saturation	After Saturation
W = weight in air, lbs.	55.	55 + (0.30 x 62.4) = 73.72
F_B = buoyant force when submerged, lbs.	62.4	62.4
$W-F_B$ = weight when submerged in water (net weight), lbs.	55 − 62.4 = − 7.4	73.72 − 62.4 = 11.32

The degree to which the factors discussed above are capable of affecting the net or stabilizing weight of a structure is illustrated by the following example:

Assume a timber crib diversion dam subject to complete submergence under normal flood flows. Materials, weights, and volumes are:

Material	Percent of Volume of the Dam	Unit Weights lbs/ft^3
Timber	12	55 in air
Timber		73 saturated
Loose stone, 30 percent voids	88	150 solid stone

Determine the net weight of one cubic yard of the dam when (1) not submerged; (2) submerged but timber not saturated; (3) submerged with timber saturated:

1. Compute cubic feet of timber, solid stone, and voids per cubic yard of dam:

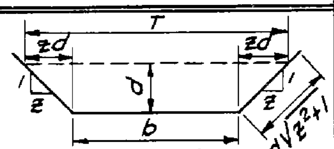
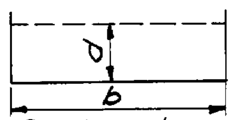
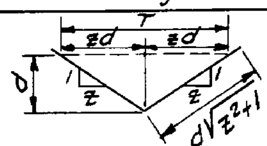
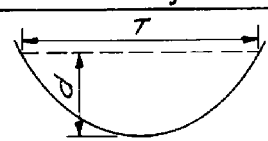
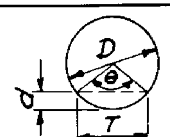
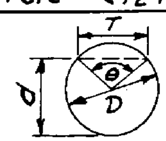
a. Timber: $0.12 \times 27 = 3.24$ ft.^{3}
b. Solid stone: $0.7 \times 0.88 \times 27 = 16.63$ ft.^{3}
c. Voids: $0.3 \times 0.88 \times 27 = \underline{7.13}$ ft.^{3}

27.00 ft.^{3}

2. Compute the net weights of one cubic yard of dam:

Material	Net Weights of Materials in lbs./cu.yd. of Dam		
	Not Submerged	Submerged	
		Timber not Saturated	Timber Saturated
Timber	3.24 x 55 = 178	3.24(55 − 62.4)= − 24	3.24(73 − 62.4)= 34
Stone	16.63 x 150 = 2494	16.63(150 − 62.4)= 1457	1457
Effective or stabilizing weight of dam per cu. yd.	= 2672	1433	1491

HYDRAULICS: ELEMENTS OF CHANNEL SECTIONS

Section	Area a	Wetted Perimeter p	Hydraulic Radius r	Top Width T
 Trapezoid	$bd+zd^2$	$b+2d\sqrt{z^2+1}$	$\frac{bd+zd^2}{b+2d\sqrt{z^2+1}}$	$b+2zd$
 Rectangle	bd	$b+2d$	$\frac{bd}{b+2d}$	b
 Triangle	zd^2	$2d\sqrt{z^2+1}$	$\frac{zd}{2\sqrt{z^2+1}}$	$2zd$
 Parabola	$\frac{2}{3}dT$	$T+\frac{8d^2}{3T}$ [1]	$\frac{2dT^2}{3T^2+8d^2}$ [1]	$\frac{3a}{2d}$
 Circle - < ½ full [2]	$\frac{D^2}{8}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$\frac{\pi D\theta}{360}$	$\frac{45D}{\pi\theta}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$
 Circle - > ½ full [3]	$\frac{D^2}{8}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$\frac{\pi D(360-\theta)}{360}$	$\frac{45D}{\pi(360-\theta)}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$

[1] Satisfactory approximation for the interval $0 < \frac{d}{T} \leqq 0.25$
When $d/T > 0.25$, use $p=\frac{1}{2}\sqrt{16d^2+T^2}+\frac{T^2}{8d}\sinh^{-1}\frac{4d}{T}$

[2] $\theta=4\sin^{-1}\sqrt{d/D}$
[3] $\theta=4\cos^{-1}\sqrt{d/D}$
} Insert θ in degrees in above equations

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-33 SHEET 1 OF 1 DATE 6-6-50

3. Fundamentals of Water Flow

3.1 Laminar and Turbulent Flow. Water flows with two distinctly different types of motion; laminar and turbulent.

When laminar flow occurs, the individual particles of water move along straight or orderly path lines. In straight conduits the path lines are straight and parallel; in irregular conduits or in passing obstacles they are orderly lines which do not intersect. With laminar motion, the mean velocity of flow varies directly with the slope of the hydraulic gradient.

In the case of turbulent flow, the water particles follow winding, irregular paths that are generally spiral in form. In addition to the main velocity in the direction of flow, there are transverse components of velocity. The mean velocity of flow varies with the square root of the slope of the hydraulic gradient.

The change from laminar to turbulent flow occurs at a velocity which is determined by the dimensions of the conduit and the viscosity of the water. In engineering practice the decision as to whether laminar or turbulent flow will occur in a given case is based on the Reynold's number value.

$$R = \frac{Lv}{\nu} \qquad (5.3\text{-}1)$$

R = Reynold's number..

L = a linear dimension of the conduit such as diameter of pipe or depth of flow.

v = mean velocity of flow.

ν (Greek nu) = kinematic viscosity.

Reynold's number is dimensionless; that is, it has the same value regardless of the system of consistent units used. The reports of various investigators indicate that in pipe flow Reynold's number values of 2000 or less characterize laminar motion, and 3000 or more turbulent motion with a transition range between these values. There are fewer reports of experiments with open flow, but it appears that Reynold's number values comparable to the above for open flow are about 500 to 1500 respectively. Reynold's number is the ratio of inertia force to viscous force and has broader significance than serving only as a criterion to distinguish between laminar and turbulent flow.

The type of motion with which water flows under different conditions has practical significance. Laminar flow is important to the hydraulic engineer because it is the type of motion with which percolation occurs. Problems dealing with the passage of water through soils, sands, gravels, or porous solids are solved by the application of the mechanics of laminar flow. Turbulent motion characterizes the flow in field hydraulic structures.

3.2 Continuity of Flow. When the discharge at a given cross section of a channel or pipe is constant with respect to time, the flow is steady. If steady flow occurs at all sections in a reach, the flow is continuous and

$$Q = a_1 v_1 = a_2 v_2 = a_3 v_3 \qquad (5.3\text{-}2)$$

Q = discharge.

a = cross-sectional area.

v = mean velocity of flow.

1,2,3 = subscripts denoting different cross sections.

Equation (5.3-2) is known as the equation of continuity. The majority of our hydraulic problems deal with cases of continuous flow.

3.3 Energy and Head. Three forms of energy are normally considered in the analysis of problems in water flow: kinetic energy, potential energy, and pressure energy.

Kinetic energy exists by virtue of the velocity of motion and amounts to $Mv^2/2$, where M is any mass and v is velocity. Since $M = W/g$, the kinetic energy is $Wv^2/2g$, and when W = 1 lb. it has the value $v^2/2g$. Note that $v^2/2g$ being composed of the following units expresses velocity head only:

$$\frac{ft^2/sec^2}{ft/sec^2} = \frac{ft^2}{sec^2} \times \frac{sec^2}{ft} = ft.$$

However, it is directly proportional to the kinetic energy of the flowing water and is derived by assuming a weight of 1 lb; therefore, it is an expression of the kinetic energy in foot pounds per pound. If time is considered, the velocity head is also an expression of foot pounds per pound per second.

Potential energy is the ability to do work because of the elevation of a mass of water with respect to some datum. A mass of weight, W, at an elevation z feet, has potential energy amounting to Wz foot pounds with respect to the datum. The elevation head, z, expresses not only a linear quantity in feet, but also energy in foot pounds per pound.

A mass of water as such does not have pressure energy. Pressure energy is acquired by contact with other masses and is, therefore, transmitted to or through the mass under consideration. The pressure head, p/w, like the velocity and elevation heads, also expresses energy in foot pounds per pound.

The relationship between the three forms of energy in pipe flow and in channel flow is shown by fig. 5.3-1. On the right in each case is shown the velocity head, pressure head, and elevation head for a stream tube at point A in section 1. On the left is shown the total head and the three separate energy heads for the section containing A. The distance from any stream tube to its energy line is the sum of pressure and velocity heads. If all stream tubes composing flow have equal energy at a given section, variations in the velocity heads of stream tubes must be balanced by equal and opposite changes in the pressure heads. Therefore, if all stream tubes are to have a common energy line at a section, two conditions must be satisfied: (1) pressure intensity must vary as a straight line in accordance with the hydrostatic law; (2) the flow must be parallel and the velocities of all stream tubes must be equal.

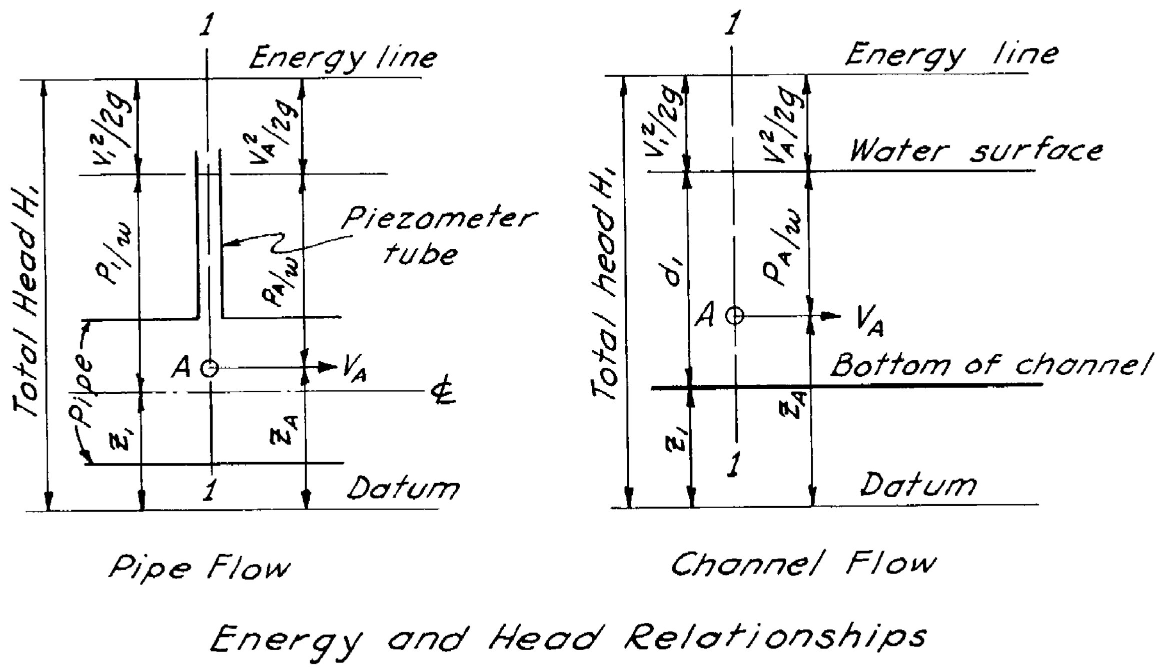


FIG. 5.3-1

In pipe flow a change in pressure head causes a uniform change in pressure intensity throughout a given cross section. Therefore, changing hydrostatic head on a pipe system does not alter the pattern of motion, and the variation in the energy of the individual stream tubes composing flow at any cross section under a given hydrostatic head results only from the unequal velocities of the stream tubes. This is illustrated by fig. 5.3-2. The pressure diagram on the vertical diameter of a pipe is shown by ABCE. The complete pressure diagram is a truncated cylinder for which each vertical section is similar to ABCE. Variation of the pressure head, h_p, would change only the ABCD portion of the pressure diagram for which the unit pressure is uniform. Furthermore, potential energy, the sum of pressure and elevation heads, with respect to any datum is constant over the cross section, since variation in pressure head is balanced by an equal and opposite variation in elevation head. Variation in the velocities of the different stream tubes accounts for the variation in the energy of flow of the stream tubes at a given section.

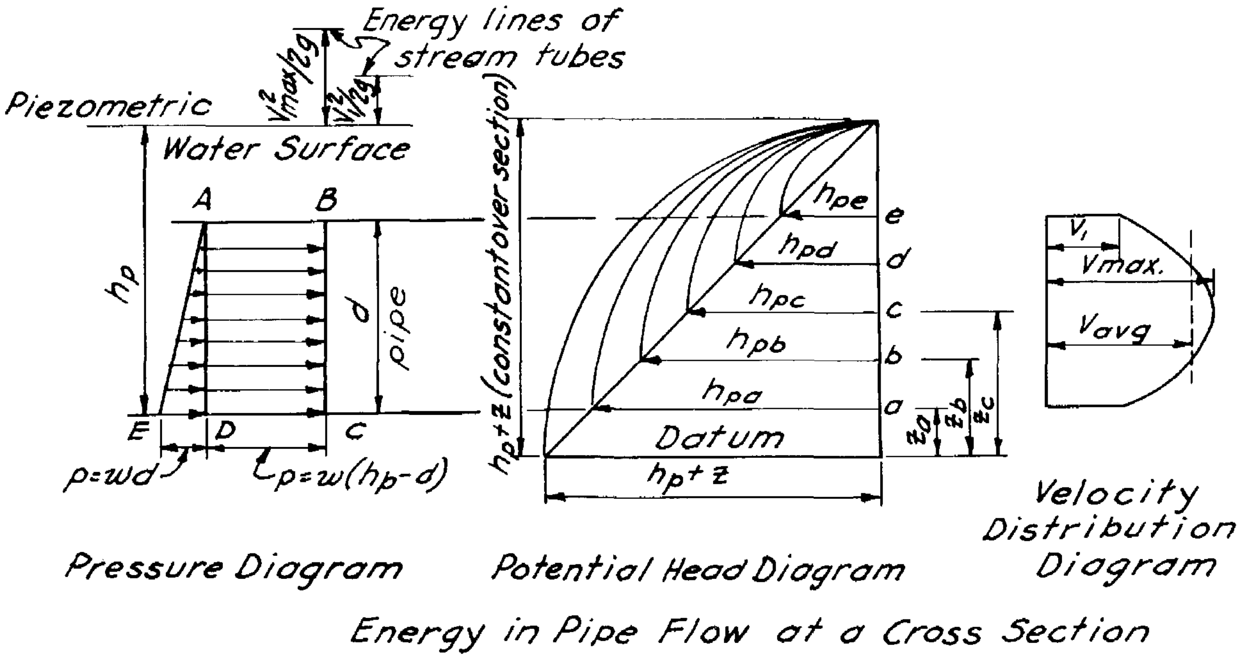


FIG. 5.3-2

In open flow, pressure at the surface is atmospheric, and internal pressure cannot be changed without altering the pattern of flow. Curvilinear flow changes the internal pressure distribution through dynamic effect and, therefore, changes the flow pattern.

If open flow is parallel, the potential energy head is constant over any cross section and only the velocity head varies from one stream tube to another. This is illustrated by fig. 5.3-3.

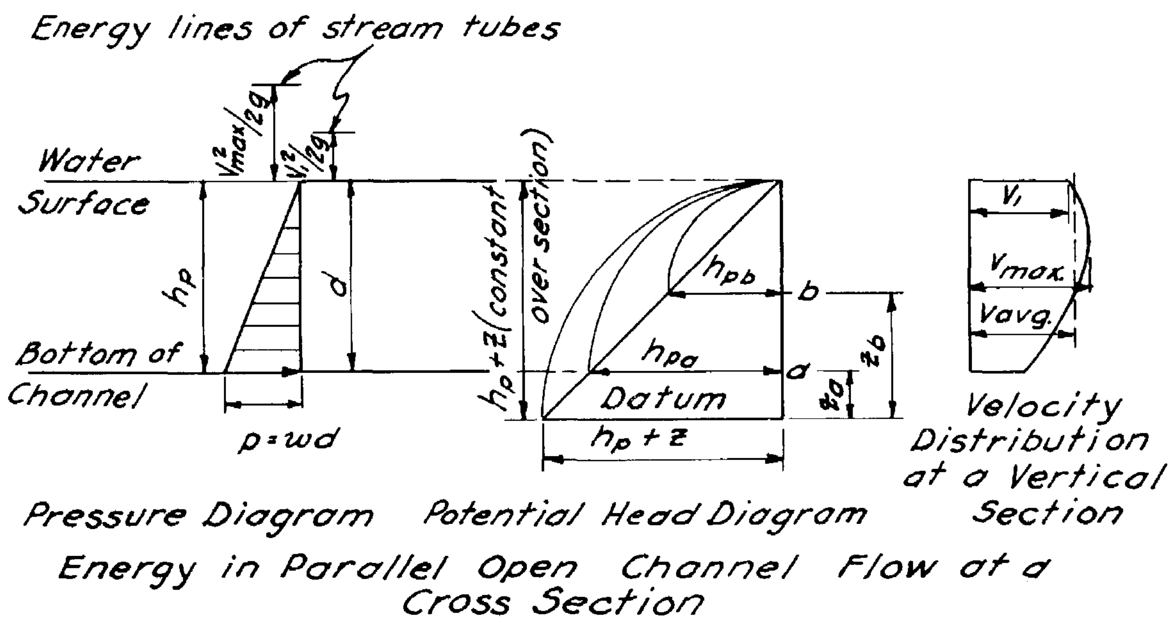


FIG. 5.3-3

The above shows that in order to obtain a total head accurately representing the mean energy of flow, it is necessary to compute a weighted mean velocity head for addition to the constant potential head at a cross section. The equation expressing the weighted mean velocity head is:

$$h_v = \alpha \frac{v^2}{2g} \qquad (5.3\text{-}3)$$

h_v = weighted mean velocity head of flow at a cross section.
v = mean velocity of flow.
g = acceleration of gravity.
α (Greek alpha) = a kinetic energy correction factor, the value of which depends upon the distribution of velocity in the cross section of flow.

A method of determining α is given on page 260 of "King's Handbook." The value of α for relatively uniform velocity distribution is 1.05 to 1.10. Wide variations in velocity such as are found in obstructed flow or irregular alignment may produce values of α of 2.0 or greater. Problems may be encountered, therefore, in which a kinetic energy correction must be applied to velocity head if computations within reasonable limits of accuracy are to be made. In the majority of cases $v^2/2g$ is accepted as a sufficiently accurate expression of velocity head.

In pipe flow problems it is common practice to measure elevation head from the datum to the center line of the pipe, the pressure head from the center line to the piezometric surface, and the velocity head from the elevation established by the pressure head. In open channel flow the elevation head is measured from the datum to the bottom of the channel, pressure head is the depth of flow, and velocity head is measured from the water surface.

3.4 Bernoulli Theorem. This theorem is the application of the law of conservation of energy to fluid flow. It may be stated as follows: In frictionless flow the sum of the kinetic energy, pressure energy, and elevation energy is equal at all sections along a stream. In practice, friction and all other energy losses must be considered and the energy equation becomes:

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2 + h_f + h_\ell \qquad (5.3\text{-}4)$$

v = mean velocity of flow.
p = unit pressure.
w = unit weight of water.
g = acceleration of gravity.
z = elevation head.
h_ℓ = all losses in head other than by friction between sections 1 and 2.
h_f = head lost by friction between sections 1 and 2.
1 and 2 denote upstream and downstream sections respectively.

The energy equation and the equation of continuity are the two basic, simultaneous equations used in solving problems in water flow.