United States Department of Agriculture

Soil Conservation Service

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TECHNICAL RELEASE NO. 66 (THIRD EDITION)
210-VI

SUBJECT: ENG - SIMPLIFIED DAM-BREACH ROUTING PROCEDURE

Purpose. To distribute the second revision of Technical Release No. 66.

Effective Date. Effective when received.

This revision establishes a common theoretical basis for the graphical and the numerical solutions of the dam-breach problem by the simplified Attenuation-Kinematic flood routing method. Thus, the mathematical models for the curvilinear and triangular hydrographs used by the method now appear in the text rather than the appendix. Also, detailed procedures for solving the models by either approach are included.

Families of curves representing the equations of the two mathematical models have replaced the single parameter curves of ES-212; thus, the user may read solution values directly from these graphs, thereby eliminating the need for adjustments. As another addition, this revision includes a method for estimating the arrival time of the peak discharge at points downstream of the dam.

Appendix A of the revision provides the documentation for the complete Attenuation-Kinematic flood routing method and dwells in greater detail, than before, on the theoretical and practical aspects of the routing method.

A series of evaluation tests of the simplified procedure revealed a systematic trend towards excessive attenuation of the routed peak discharge with distance. This evaluation suggests that the application of the procedure be limited to situations in which the value of the parameter k^* is less than one.

PAUL M. HOWARD
Deputy Chief
for Technology

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U.S. DEPARTMENT OF AGRICULTURE
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TECHNICAL RELEASE NO. 66
(THIRD EDITION)

SIMPLIFIED DAM-BREACH ROUTING PROCEDURE

U. S. DEPARTMENT OF AGRICULTURE

SOIL CONSERVATION SERVICE

ENGINEERING DIVISION

DESIGN UNIT

SEPTEMBER 1985

Preface

Recent dam failures have underscored the need to be able to predict flood stages downstream from breached dams. This technical release presents a simplified procedure for making these predictions. These predictions would supply the information needed to evaluate the safety hazards of existing and planned dams if they should breach, to assist planning for development in downstream areas, and to help civil defense and rescue agencies prepare for dam failures.

The Soil Conservation Service, in its mission of soil and water conservation, examines a large number of potential dam sites for landowners and project sponsors. Generally, these dams are of small height and storage; thus, a potential dam breach would have a small impact on the flood plain areas beyond a relatively short distance downstream (often five miles or less). Therefore, a breach routing method is needed that is easy to use and provides quick and accurate information.

In developing this technical release, many routing methods were considered. The selected Att-Kin procedure best meets the SCS need.

The purpose of this revision is to establish a common theoretical basis for both the graphical and numerical solutions of the dam-breach problem. Thus, the mathematical models for the curvilinear and triangular hydrographs used by the method now appear in the text rather than the appendix. Also, detailed procedures for solving the models by either approach are included.

Families of curves representing the equations of the two mathematical models have replaced the indirect, graphical procedure which used the two single parameter curves on ES-212 and an empirical numerical technique. Another addition to this revision is a method for estimating the arrival time of the peak discharge at points downstream of the dam.

This revision also clarifies the theoretical and practical basis of the parent Att-Kin routing method. The presentation of the Att-Kin method in Appendix A is preceded by a brief presentation of major concepts underlying its formulation, as well as, a discussion of similarities and departures from other simplified flood routing methods. These provide a framework for a rational evaluation of the Att-Kin method through comparison to conventional alternatives and, by extension, for an assessment of the advantages and the drawbacks in its application.

John A. Brevard and Fred D. Theurer prepared the original technical release. Dr. Theurer with the assistance of George H. Comer developed the routing procedure.

George Kalkanis, Civil Engineer, Design Unit, Engineering Division prepared this revision. William H. Merkel, Hydraulic Engineer, Hydrology Unit, Engineering Division consulted with Dr. Kalkanis during its preparation.

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Tables

Appendix A - The Att-Kin Flood Routing Method

Nomenclature

A	≡	flow area, ft^2
$A_{i,j}$	≡	flow area associated with discharge Q_i at section j, ft^2
$A_{i,j+1}$	≡	flow area associated with discharge Q_i at section j+1, ft^2
c	≡	dQ/dA, speed of propagation of discharge Q, ft/sec
C	≡	coefficient of proportionality
d	≡	depth of flow, ft
d_o	≡	maximum depth of flow associated with discharge Q_o, ft
g	≡	acceleration of gravity, ft/sec^2
k	≡	coefficient in the discharge-valley storage relationship
k_o	≡	coefficient in the discharge-flow area relationship
k_s	≡	coefficient in the discharge-valley storage relationship used in the "storage" routing submodel
L_j	≡	length of subreach j, ft
L_{j-1}	≡	length of subreach j-1, ft
m	≡	exponent in the discharge-valley storage relationship
m_s	≡	exponent in the discharge-valley storage relationship used in the "storage" routing submodel
n	=	$\frac{N}{2}$ when N is an even number and $\frac{N+1}{2}$ when N is odd
N	≡	number of paired values of Q_i, $S_{i,j}$
Q	≡	discharge, cfs
Q_i	≡	a particular discharge in an array of discharges, cfs
Q_1	≡	instantaneous outflow discharge from a reach, cfs
$Q_{1,1}$	≡	instantaneous outflow discharge from a reach at time t_1, cfs
$Q_{1,2}$	≡	instantaneous outflow discharge from a reach at time t_2, cfs

$\overline{Q}_1$ = $\frac{Q_{1,1} + Q_{1,2}}{2}$, mean outflow discharge from a reach during the time interval $\Delta t = t_2 - t_1$, cfs

$Q_{1,i}$ ≡ outflow discharge from the reach at time $t_{1,i}$ determined by the storage submodel, as well as, at times $t_{1,i} + \delta t_i$ and $t_{1,i} + \delta t_{1,i}$ determined by the Att-Kin model, cfs

Q_2 ≡ instantaneous inflow discharge into a reach, cfs

$Q_{2,1}$ ≡ instantaneous inflow discharge into a reach at time t_1, cfs

$Q_{2,2}$ ≡ instantaneous inflow discharge into a reach at time t_2, cfs

$\overline{Q}_2$ = $\frac{Q_{2,1} + Q_{2,2}}{2}$, mean inflow discharge into a reach during the time interval $\Delta t = t_2 - t_1$, cfs

Q_I ≡ maximum discharge of inflow hydrograph, cfs

Q_{max} ≡ maximum discharge of breach hydrograph, cfs

Q_o ≡ maximum outflow discharge from a reach, cfs

Q^* = $\frac{Q_o}{Q_I}$

r ≡ hydraulic radius, ft

S ≡ valley storage, ft^3

S_g ≡ longitudinal slope of floodway floor

$S_{i,j}$ ≡ valley storage in subreach j associated with outflow discharge Q_i, ft^3

$S_{i,j-1}$ ≡ valley storage in subreach j-1 associated with outflow discharge Q_i, ft^3

$S_{i,d,j}$ ≡ off-channel valley storage between section j and j+1 associated with discharge Q_i, ft^3

S_I ≡ valley storage in the reach associated with Q_I through the discharge-storage relationship used by the Att-Kin routing model, ft^3

S_n	$\equiv$	valley storage in the reach satisfying continuity at time t_o , ft^3
S_o	$\equiv$	valley storage in the reach at time t_o associated with Q_o from discharge-valley storage relationship, ft^3
$S_{1,i}$	$\equiv$	valley storage in the reach at time $t_{1,i}$ determined by the storage routing submodel, ft^3
$S_{1,i-1}$	$\equiv$	valley storage in the reach at time $t_{1,i-1}$ determined by the storage routing submodel, ft^3
$S(t)$	$\equiv$	valley storage in the reach at time t and associated with Q(t) through the discharge-storage relationship used by the storage routing submodel, ft^3
t	$\equiv$	time
t_1	$\equiv$	time at the beginning of routing time interval
t_2	$\equiv$	time at the end of routing time interval
t_I	$\equiv$	time to peak inflow discharge Q_I into the reach
t_o	$\equiv$	time to peak outflow discharge Q_o from the reach
t_s	$\equiv$	time to peak outflow discharge Q_o from the reach determined by the storage routing submodel
$t_{1,i-1}$	$\equiv$	time at the beginning of computational interval in the storage routing submodel
$t_{1,i}$	$\equiv$	time at the end of computational interval in the storage routing submodel
t^*	=	$t \dfrac{Q_I}{V_I}$
t_o^*	=	$t_o \dfrac{Q_I}{V_I}$
V	=	$\dfrac{Q}{A}$, average flow velocity through any valley cross-section, ft/sec

V_d ≡ valley storage in subreach attributed to the kinematic distortion of the outflow hydrograph produced by the storage routing submodel, ft^3

V_I ≡ total volume of water under the breach hydrograph, i.e., excluding the volume of base flow, ft^3

V_o ≡ volume of net outflow from the reach to time t_o, ft^3

V_s ≡ valley storage in subreach associated with Q_o and t_s determined by the storage routing submodel, ft^3

V_t ≡ volume of water inflow into subreach during the time interval $t_o - t_s$, ft^3

θ ≡ angle of the tangent through the point[Q, A] on the curve representing the discharge-flow area relationship in a given valley cross-section, $= \tan^{-1}(c)$

δt_i ≡ time lag of outflow discharge $Q_{1,i}$ due to kinematic distortion

δt_o ≡ time lag of outflow discharge $Q_{1,i}$ due to kinematic translation

$\delta t_{1,i}$ ≡ total time lag for discharge $Q_{1,i}$ between hydrographs routed by the storage submodel and the Att-Kin model

TECHNICAL RELEASE

NUMBER 66 (THIRD EDITION)

SIMPLIFIED DAM-BREACH ROUTING PROCEDURE

INTRODUCTION

This technical release presents a method for estimating key characteristics of the floodwave generated by the sudden breaching of a dam. These characteristics are the peak flood flow Q_o at a predefined location in the path of the floodwave, the associated maximum depth of flow d_o, and the time lapse t_o between the breaching of the dam and the occurrence of the above two extremes. The expressions for these three dependent variables, which are furnished by the solution of an empirical mathematical model discussed later, are functions of the independent variable representing distance from the dam, as well as, certain parameters defining the breach hydrograph and describing pertinent hydraulic characteristics of the valley downstream. These parameters are determined as follows:

BREACH HYDROGRAPH

The breach hydrograph, as all hydrographs, is completely defined by its peak discharge Q_{max}, its total volume V_I, and its shape.

Regarding shape, the method postulates that the breach hydrograph is a continuous decaying function of time of either triangular or curvilinear shape. More precisely, the decay in the latter case is exponential. The rule of selecting the applicable shape in a given situation is based on the anticipated flow regime in the valley subreach immediately below the dam. Thus, if the expected flow in the subreach is supercritical, the applicable shape is triangular; otherwise, the appropriate hydrograph shape is curvilinear.

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The reasoning behind the rule is that, if the flow in the reach immediately below the breached dam is subcritical, the tailwater will submerge the breach at some outflow, thereby retarding the total flow from the breach. Then, the breach hydrograph will assume a curvilinear shape. Conversely, if the flow is supercritical, the tailwater will be lower, thereby impeding the flow less than before. Then, the shape of the hydrograph will be triangular.

Q_{max} may be determined by methods that are process based, using scientific procedures for erosion, sediment transport, and hydraulics, or based on empirical relationships derived from analysis of recorded actual dam failure data. The minimum required peak breach discharge is established by SCS policy.

Figure 8 shows a plot of paired data of maximum breach discharge and depth of water for recorded dam failures. Also shown is a line almost enveloping the data which, if used, would predict a rather conservative maximum breach discharge.

The total volume V_I is the sum of the volume of water being stored behind the dam at the time of breaching and of the part of the storm hydrograph still approaching the reservoir from upstream. Thus, in considering the event of dam breaching without storm inflow, the second component of V_I is set equal to zero.

HYDRAULICS

The second key component of the simplified Att-Kin method consists of hydraulic characteristics of the valley downstream of the dam. For a given valley reach a single-valued relation of discharge and valley storage must be determined. This relation may be determined from stream gage data or water surface profiles obtained from unsteady, steady non-uniform, or steady uniform flow computations. Of course, the accuracy of the discharge-storage relation depends on the method used and the level of detail of the cross-section data collected.

In order to determine the discharge-storage relation for a reach, cross-section data, roughness coefficients, and reach lengths are needed. The detail of this data gathering and analysis is dependent on the purpose for doing the breach routing and on such factors as variation of valley cross-section (and slope) and floodplain land use. Methods for computing water surface profiles for valley cross-sections are contained in many hydraulics texts and also in SCS TR-61 and SCS NEH Section 4.

Profiles should be computed for a range of discharges up to the maximum discharge expected from the breach.

The discharge-valley storage relation should reflect total valley storage between cross sections for specific discharges. For a steady discharge, the associated valley storage is the main valley storage and the off-channel or "dead" valley storage between cross-sections. The off-channel valley storage is any valley storage not accounted for in the flow area determinations, for example, draws and channels upstream of confluences.

Following the development of the rating curves at key valley cross-sections, the total valley reach is divided into subreaches. Each subreach is bounded by the valley cross-section at the dam and by one of the remaining downstream cross-sections. Reach 1 is bounded by cross-sections 1 and 2, reach 2 by cross-sections 1 and 3,.., and reach n_1-1 by cross-sections 1 and n_1; n_1 being the number of cross-sections.

The discharge-valley storage values defining the discharge-valley storage relation for each subreach must be determined. The equation determining valley storage $S_{i,j}$ is,

$$S_{i,j} = S_{i,j-1} + \frac{(A_{i,j} + A_{i,j+1})}{2} (L_j - L_{j-1}) + S_{i,d,j} \qquad (1)$$

in which,

$A_{i,j}$	≡	flow-area associated with discharge Q_i at section j, ft^2
$A_{i,j+1}$	≡	flow area associated with discharge Q_i at section j+1, ft^2

$S_{i,j}$	≡	valley storage in subreach j associated with discharge Q_i, ft^3
$S_{i,j-1}$	≡	valley storage in subreach j-1 associated with discharge Q_i, ft^3
L_j	≡	length of subreach j, ft
L_{j-1}	≡	length of subreach j-1, ft
$S_{i,d,j}$	≡	off-channel storage between section j and j+1 associated with discharge Q_i, ft^3
Q_i	≡	a particular discharge in an array of discharges, cfs

By definition,

$$S_{i,0} = L_1 = 0 \quad \text{for all } i's$$

The discharge-valley storage relation for a subreach is represented by the equation

$$Q_i = k_j \, (S_{i,j})^{m_j} \qquad (2)$$

in which,

j	≡	1 to n_1-1
i	≡	1 to N
n_1	≡	number of valley cross-sections
N	≡	number of water surface profiles

Ordinarily, the value of n_1 is predetermined, but the user selects the value of N, preferably, an even number and as large as convenient.

Taking the logarithm of both sides of equation (2) produces the equation,

$$\log Q_i = \log k_j + m_j \log(S_{i,j}) \qquad (3)$$

from which the values of k_j and m_j are determined through application of a linear regression technique on paired values of log (Q_i) and log ($S_{i,j}$) for subreach j. The most prominent and widely used technique for the purpose is the method of "least squares", whose coded versions may be found in most ADP libraries. However, for sake of completeness, the rather computationally simple "method of averages" (Smith, Gale, and Neelley 1956) is given in Appendix B.

An observation of practical significance is that, in the particular case of uniform flow, the equation relating the valley storage to the valley cross-section is,

$$S = A\ L \quad \text{for all } i\text{'s and } j\text{'s} \tag{4}$$

where

$A \equiv$ cross-section flow area, ft^2

$L \equiv$ reach length, ft

It follows that

$$Q = kS^m = k(AL)^m = k\ L^m\ A^m \tag{5}$$

defining

$$k_o = k\ L^m$$

and substituting into equation (5) and from equation (4)

$$Q = k_o\ A^m = \frac{k_o}{L^m}\ S^m \tag{5A}$$

The values of k_o and m in equation (5A), which for uniform flow (and no dead storage) is a substitute for equation (2), can be determined directly from paired values of Q and A computed by any reputable friction formula.

THE MATHEMATICAL MODEL

A feature shared by most simplified methods solving the dam breach problem is that the breach hydrograph, which normally is a boundary condition, constitutes an integral part of the mathematical model simulating response. Thus, the recognition of two distinct shapes of the breach hydrograph imposes the necessity of two distinct mathematical models. The selection of the applicable hydrograph shape becomes, in essence, the criterion for selecting the applicable mathematical model.

The development of the model evolved through a process of successive modifications and adjustments of a set of equations. The approach was identical for both curvilinear and triangular hydrograph models so, in the discussion that follows, the term "model" is representative of both.

The process consisted of fitting pertinent dam breach data, measured or computed, to mathematical expressions containing the variables defined on the following pages; and using judicious adjustments, until satisfactory levels of closeness were achieved. The measured data used in the process was from actual dam-breach events and physical model studies (Theurer and Comer 1979). The computed data was generated by combining results from two mathematical flood routing models; one was hydraulic, based on the method of characteristics (Theurer 1975), and the other was hydrologic, based on the Attenuation-Kinematic method, or in brief, Att-Kin method. The Att-Kin method, which shares certain features with the simplified Att-Kin method, is described in Appendix A.

A major shared feature is the steady flow assumption underlying the derivation of equation (2) in the simplified Att-Kin method and equation (A-7) in the Att-Kin method. This assumption, though essential to its formulation, is not peculiar to the Att-Kin method. It is used by practically all hydrologic methods of routing.

Had the development of the mathematical model been purely empirical, the process of determining the form of both independent relationships would have been wholly statistical. However, in the approach used, one relationship was predefined during the initial stage of development. The predefined form of the relationship was,

$$t_o = m\,[(S_I - S_o)/(Q_I - Q_o)] \quad (6)$$

in which,

Q_I ≡ peak inflow discharge

Q_o ≡ peak outflow discharge from the subreach

t_o ≡ time to peak outflow discharge Q_o

S_I ≡ valley storage in the subreach associated with Q_I

S_o ≡ valley storage in the subreach associated with Q_o

Equation (6) is a modified version of a characteristic equation of the Att-Kin model. It appears in Appendix A as equation (A-21). The two-fold modification involved the elimination of the term, t_I, for hydrographs rising suddenly to the peak and multiplication of the bracket on the right-hand side of the equation by m. The justification for the latter was the observation that, under the dry-bed initial condition prescribed by the method, the celerity of the flood wave at the leading tip cannot exceed the mean flow velocity in the valley reach.

Initially, equation (6) was common to both hydrograph shapes. A second equation for each shape was developed empirically from data. In the process, equation (6) underwent additional adjustments and, eventually, assumed a different form for each submodel. The final versions are represented below in dimensionless notation by equation (8) for the curvilinear hydrograph and equation (11) for the triangular.

Curvilinear Hydrograph

$$Q_i = Q_I e^{-t^*} \quad (7)$$

$$t_o^* = m\,[\,Q^{*^{-\frac{1}{m}}} - 1\,] \quad (8)$$

$$k^* = \{[1 - e^{-t_o^*}] + \frac{1}{2}[\,Q^{*^2} \ln Q^*]\}^m \{\frac{1}{Q^*}\} \quad (9)$$

Triangular Hydrograph

$$Q_i = Q_I \left[1 - \frac{t^*}{2} \right] \quad \text{for} \quad t^* < 2 \tag{10A}$$

$$Q_i = 0 \quad \text{for} \quad t^* > 2 \tag{10B}$$

$$t_o^* = m\,(1 + Q^*)\left[Q^{*\,-\frac{1}{m}} - 1\right] \tag{11}$$

$$k^* = \left\{ t_o^* \left[1 - \frac{1}{4} t_o^* \right] - Q^{*2} \; (1 - Q^*) \right\}^m \left\{ \frac{1}{Q^*} \right\}$$

$$\text{for} \quad t_o^* < 2 \tag{12A}$$

$$k^* = \left[1 - Q^{*2} \; (1 - Q^*) \right]^m \left[\frac{1}{Q^*} \right] \quad \text{for} \quad t_o^* > 2 \tag{12B}$$

and in which by definition,

$$Q_I \equiv Q_{max} \tag{13}$$

$$Q^* \equiv \frac{Q_o}{Q_I} \tag{14}$$

$$t^* \equiv \frac{t\,Q_I}{V_I} \tag{15}$$

$$t_o^* \equiv \frac{t_o\,Q_I}{V_I} \tag{15A}$$

$$k^* \equiv \left(\frac{S_I}{V_I}\right)^m \tag{16}$$

Since for a given j, equation (2) becomes $Q_I = k\,S_I^{\,m}$

$$S_I^{\,m} = \frac{Q_I}{k}$$

After substitution of $\frac{Q_I}{k}$ for $S_I^{\,m}$, equation (16) becomes,

$$k^* \equiv \frac{Q_I}{k\,V_I^m} \tag{16A}$$

The first equation in either set, i.e., equation (7) and equation (10), describes the breach hydrograph associated with the model. These equations are not used in the solution because the pertinent information they contain has already been incorporated in the remaining equations of the associated model. Equations (8) and (9) form the mathematical model for the curvilinear shape hydrograph and equations (11) and (12A) or (12B) for the triangular shape hydrograph. Each model consists of a set of two independent equations containing four unknowns; k^*, t_o^*, Q^*, and m. For each model, if two of the variables are predetermined, the other two may be determined. Normally, m is determined from the discharge-storage relation, equation (2), and k^* is determined from equation (16A), then t_o^* and Q^* may be solved for directly.

In summary, the breach routing solution consists of solving equation (2), equation (16A), and either equations (8) and (9) or equations (11) and (12A) or (12B).

An important observation is that while the time t_o and the discharge Q_o appear explicitly in the expressions for the nondimensional parameters, the distance L does not. In the general case of nonuniform flow represented by equation (16), the value of L, as indicated clearly by equations (1) and (2), is reflected indirectly in the value of S_I.

The only time L appears explicitly is when the flow is uniform; for then,

$$S_I = A_I L \tag{17}$$

$A_I \equiv$ flow area associated with peak inflow discharge Q_I

Substituting the right-hand side of equation (17) for S_I in equation (16) gives,

$$k^* = \left(\frac{A_I L}{V_I}\right)^m \tag{18}$$

Since in a given situation m, A_I, and V_I are fixed variables, the dimensionless parameter k^* is a simple exponential function of L.

The final observation is that d_o, the maximum depth of inundation, does not appear in the mathematical model because it is not determined directly by it. Its value is determined separately from the value of Q_o. The determination is done by means of the rating curve at the target cross-section developed, as mentioned earlier, from water surface profile computations.

APPLICATION OF THE SIMPLIFIED ATT-KIN METHOD

Empirical testing of the method for accuracy of predictions is hampered by the absence of reliable data from actual dam failures. So, the benchmark information has to be generated, artificially, from solutions of more sophisticated mathematical models. The problem is that practically all such models are incompatible with the simplified Att-Kin model. For instance, the NWS DAMBRK model, perhaps the most efficient in its class, will not admit breach hydrographs exhibiting an instantaneous rise to the peak, a built-in characteristic of the simplified Att-Kin model. Moreover, it cannot be solved for an initial dry-bed condition, another characteristic peculiar to that model.

The foregoing notwithstanding, the accuracy of the method has been evaluated through a comparison test of its results with data generated under slightly modified conditions by the NWS DAMBRK model. Even though limited in scope, the test confirmed the predicted tendency of the simplified Att-Kin model, consistent with the dry-bed assumption, of a higher than normal rate of attenuation of the peak discharge with distance. Confirmation of suspected questionable behavior suggested limiting the method's application to situations in which the value of the parameter k^* is smaller than or equal to 1.0. In the domain beyond that limit, the method may be used with caution, that is, with the understanding that predicted values are good only for qualitative assessments of potential hazards.

Where an obstruction, such as a road embankment, stores a significant portion of the flow from the dam, the obstruction controls the flow downstream; therefore, the routing procedure is not applicable downstream of the obstruction.

In the simplified Att-Kin procedure, the storage-discharge curve described by the calculated m and k values is normally used to represent the momentum equation. However, where the m value is greater than three or less than one, the storage-discharge relation is not a good substitution for the momentum equation. Values of m outside of the range $1 < m < 3$ represent conditions which are physically unrealistic for open channel flow. Analysis shows that for $m < 1$, pressure flow exists and for $m = 3$, laminar flow exists. Thus, an m value less than one or greater than three may be a good representation of the storage-discharge curve, but it is not a good representation of the momentum equation.

Thus, where the m value associated with the storage-discharge relation is outside of the range of one to three, other routing procedures should be used. The applicable range for m values is shown in figures 4 through 7.

PROBLEM STATEMENT AND PROCEDURE

Problem Statement

Given: Q_{max}, V_I,
geometric and roughness data for valley cross-sections,
and associated distances from the dam;

Find : Q_o, d_o, and t_o at the valley cross-section located at distance L downstream from the dam.

Procedure

1. Use given data to develop depth of flow versus area curves for all cross-sections.

2. Perform water surface profile computations for an array of discharges containing Q_{max}, under the assumptions of subcritical flow conditions in the valley and uniform flow at its downstream end section. If the computation terminates normally, go to next step. If not, repeat it starting

at the upstream end section with flow at critical depth. If this termination is normal, go to next step. If not, terminate the computation; the given set of data is not manageable by the method.

3. Use data computed in step 2 to develop a table relating discharge with depth of flow and area at each cross-section.

4. Determine shape of the breach hydrograph.
   Use data for the valley cross-section immediately downstream from the dam, generated in the steps above, to determine flow area A and top width T associated with Q_{max}.

   Compute the square of the associated critical flow discharge at that cross-section from,

$$Q_{c,d}^{2} = \frac{g A^{3}}{T}$$

   The shape of the breach hydrograph is triangular if,

$$\frac{Q_{max}}{Q_{c,d}} > 1$$

   Otherwise, the shape of the hydrograph is curvilinear.

5. Use the data from steps 1 and 3 to generate area-discharge relationships and use them next to compute the associated valley storage for each subreach from equation (1).

6. Compute values of k and m for each valley reach, using paired sets of Q and S data from step 5 and the procedure in Appendix B or equivalent alternatives.

7. Set $Q_I = Q_{max}$ and compute k^* from equation (16A).

8. Use the mathematical model for the hydrograph shape selected in step 4 to compute t_o^* and Q^*.

Solve the applicable mathematical model graphically using the curves on figures 4 through 7, pages 31 to 37. Figures 4 and 5 pertain to curvilinear breach hydrographs and figures 6 and 7 to triangular ones.

In a given situation, the shape of the hydrograph and the values of m and k^* are known. The applicable figure is then entered with the value of k^*, and a line is drawn vertically upward. The coordinates on the Q^*, t_o^* plane of the point of intersection of that line with the curve labeled m are the solution values of the two dependent variables.

An alternative to the graphical method outlined above is to solve the applicable mathematical model numerically. Since neither model affords a direct solution in terms of k^*, an iterative technique has to be used. The most efficient for the purpose is one iterating about the values of Q^* until the value of k^* computed by the model is about equal to that from step 7.

9. Compute the maximum flood discharge Q_o at the target cross-section from,

$$Q_o = Q^* Q_I$$

10. Determine d_o, maximum depth of flooding at the target cross-section, from the rating curve developed in step 3.

11. Compute the time to peak outflow Q_o from,

$$t_o = t_o^* \frac{V_I}{Q_I}$$

EXAMPLE 1

Given:

Q_{max} = 35,000 cfs

Reservoir storage volume is 450 ac-ft

No storm inflow

The depth of flow versus discharge curves in figure 1

The depth of flow versus flow area curves in figure 2

The two valley cross-sections below;

section 1; at the dam

section 2; 2,500 feet downstream of dam

Determine:

The maximum depth of flow at valley cross-section 2 and the associated time of occurrence.

Solution:

The information normally generated in steps 1 through 3 of the procedure is given. The results from step 1 are shown on figure 2, and those from steps 2 and 3 are shown on figure 1. Thus, the solution begins with step 4 of the procedure.

4. Determine the shape of the breach hydrograph:

   a. Determine the depth of flow and the cross-section area at section 1 for Q_{max} = 35,000 cfs, assuming the dam does not exist.

      From figure 1 and for section 1, read the depth of flow, d_1 = 24.5 ft. From figure 2 and for d_1 = 24.5 ft, read A_1 = 6,850 ft^2.

b. Determine the critical discharge, $Q_{c,d}$, for the depth of flow at section 1.

Assuming for this example that T = 600 ft when d_1 = 24.5 ft,

$$Q^2_{c,d} = \frac{gA^3}{T} = \frac{g(6,850)^3}{600} = 1.72 \times 10^{10} \text{ ft}^6/\text{ sec}^2$$

$$Q_{c,d} = 131,000 \text{ cfs}$$

Then,

$$\frac{Q_{max}}{Q_{c,d}} = \frac{35,000}{131,000} = 0.27 < 1$$

Thus, the flow immediately downstream from the dam is subcritical, and the breach hydrograph is curvilinear.

5. Determine the flow areas at each valley cross-section and the valley storage in each valley subreach for the number of discharges necessary to define adequately the discharge versus valley storage relationship. Sample calculations are shown for 0.25 Q_{max}, (Q = 8,750 cfs).

a. Section 1, (at the dam); i = 1, J = 0

From figure 1 and for Q = 8,750 cfs, read d_1 = 16.5 ft.
From figure 2 and for d_1 = 16.5 ft, read A_1 = 2,500 ft^2.

b. Section 2; i = 1, J = 1

From figure 1 and for Q = 8,750 cfs, read d_2 = 12.6 ft.
From figure 2 and for d_2 = 12.6 ft, read A_2 = 2,400 ft^2.
From equation (1),

$$S_{1,1} = S_{1,0} + \frac{(A_1 + A_2)}{2} (L_1 - L_0) = 0 + \left(\frac{2,500 + 2,400}{2}\right)(2,500 - 0)$$

$$S_{1,1} = 6.1 \times 10^6 \text{ ft}^3$$

Table 1.--Discharge-valley storage data for example 1.

$\frac{Q}{Q_{max}}$	Q	Sect. 1	Sect. 2	Reach 1
		A_1	A_2	S_1
	cfs	ft^2	ft^2	ft^3
0.25	8,750	2,500	2,400	$6.1x10^6$
0.50	17,500	4,200	4,000	$10.3x10^6$
0.75	26,250	5,500	5,300	$13.5x10^6$
1.0	35,000	6,850	6,400	$16.6x10^6$

6. Compute the values of k and m for subreach 1:

From equations (B-2) and (B-3) and for the paired [Q, S_1] data in table 1, solve for m and k. Since this example uses four pairs of Q and S_1 values, $N = 4$ and $n = \frac{4}{2} = 2$.

$$m = \frac{[4 - 2][\log 8{,}750 + \log 17{,}500] - 2[\log 26{,}250 + \log 35{,}000]}{[4 - 2][\log 6.1x10^6 + \log 10.3x10^6] - 2[\log 13.5x10^6 + \log 16.6x10^6]}$$

$m = 1.41$

$$k = \log^{-1}\left[\frac{\log 8{,}750 + \log 17{,}500 - 1.41(\log 6.1x10^6 + \log 10.3x10^6)}{2}\right]$$

$k = 2.32 \times 10^{-6}$

7. Compute k^*:

From equation (16A) and for $Q_I = Q_{max} = 35{,}000$ cfs,

$$k^* = \frac{Q_I}{k\ V_I^m} = \frac{35{,}000}{2.32 \times 10^{-6}[450(43{,}560)]^{1.41}}$$

$k^* = 0.788$

8. Determine Q^* and t_o^*:
From figure 4 and for [m = 1.41, k^* = 0.788], read:

$$Q^* = 0.495, \quad t_o^* = 0.91$$

Check results by comparison to those from numerical solution:
From equation (8) and for Q^* = 0.495,

$$t_o^* = 0.912$$

From equation (9) and [Q^* = 0.495, t_o^* = 0.912],

$$k^* = 0.786$$

9. Determine the maximum flood discharge at section 2:
From equation (14) and [Q_I = 35,000 cfs, Q^* = 0.495],

$$Q_o = Q_I\,Q^* = 35{,}000 \times 0.495 = 17{,}000 \text{ cfs}$$

10. Determine maximum depth of flooding at section 2:
From figure 1 and for Q_o = 17,000 cfs,

$$d_o = 16.1 \text{ feet}$$

11. Compute time of occurrence of maximum depth of flooding at section 2:

$$\frac{V_I}{Q_I} = \frac{450 \times 43{,}560}{35{,}000 \times 3{,}600} = .156 \text{ hrs}$$

$$t_o = t_o^* \frac{V_I}{Q_I} = 0.912 \times 0.156 = 0.142 \text{ hrs} = 8.5 \text{ min}$$

EXAMPLE 2

Given:

Q_{max} = 127,000 cfs

Breach with storm inflow

Reservoir storage volume before the storm is 2000 ac-ft

The total volume of the storm runoff is 1000 ac-ft

The depth of flow versus flow-area curves in figure 2

The depth of flow versus discharge curves in figure 3, for the three valley cross-sections located as follows:

section 1; at the dam

section 2; 4,000 feet downstream of dam

section 3; 15,000 feet downstream of dam

Determine:

The maximum depth of flow at section 2 and at section 3 and associated times of occurrence.

Solution:

The given input data furnishes the information which is normally developed in steps 1 through 3 of the solution procedure. So, the solution starts, again, with step 4 of the procedure.

4. Determine the shape of the breach hydrograph:

   a. Determine the depth of flow and the cross-section area at section 1 for $Q_I = Q_{max}$ = 127,000 cfs, assuming the dam does not exist.
   From figure 3 and for section 1, read the depth of flow,
   d_1 = 20.8 ft.
   From figure 2 and for d_1 = 20.8 ft, read A = 4,650 ft^2.

   b. Determine the critical discharge, $Q_{c,d}$, for the depth of flow at section 1, assuming for this example that T = 575 ft, when d_1 = 20.8 ft,

$$Q^2_{c,d} = \frac{gA^3}{T} = \frac{g(4650)^3}{575} = 5.62 \times 10^9 \ ft^6/sec^2$$

$$Q_{c,d} = 75,000 \ cfs$$

Then,

$$\frac{Q_I}{Q_{c,d}} = \frac{127,000}{75,000} = 1.69 > 1$$

Thus, flow immediately downstream from the dam is supercritical, and the breach hydrograph is triangular.

5. Determine the flow areas and valley storages at each section for the discharge values necessary to define adequately the discharge versus valley storage relationship. In this example, discharges of 0.2, 0.4, 0.6, 0.8, and 1 of Q_{max} and associated valley storages are used to define the discharge versus valley storage relationship.

   Sample calculations are shown for 0.2 x Q_{max} (Q_1 = 25,400 cfs), i.e., i = 1.

   a. Section 1, (at the dam); i = 1, j = 0

      From figure 3 and for Q = 25,400 cfs, read d_1 = 12.1 ft.
      From figure 2 and for d_1 = 12.1 ft, read A_1 = 1,150 ft^2.

   b. Section 2; i = 1, j = 1

      From figure 3 and for Q = 25,400 cfs, read d_2 = 9.1 ft.
      From figure 2 and for d_2 = 9.1 ft, read A_2 = 1,400 ft^2.
      From equation (1),

$$S_{1,1} = S_{1,0} + \left(\frac{A_1 + A_2}{2}\right)(L_2 - L_1)$$

$$S_{1,1} = 0 + \left(\frac{1,150 + 1400}{2}\right)(4,000 - 0) = 5.1 \times 10^6 \ ft^3$$

c. Section 3; $i = 1$, $j = 2$

From figure 3 and for Q = 25,400 cfs, read d_3 = 7.2 ft.
From figure 2 and for d_3 = 7.2 ft, read A_3 = 1,250 ft^2.
From equation (1),

$$S_{1,2} = S_{1,1} + \left(\frac{A_2 + A_3}{2}\right)(L_3 - L_2)$$

$$S_{1,2} = 5.1 \times 10^6 + \left(\frac{1,400 + 1,250}{2}\right)(15,000 - 4,000)$$

$$S_{1,2} = 19.7 \times 10^6 \ ft^3$$

Table 2.--Discharge-valley storage data for example 2.

$\frac{Q}{Q_{max}}$	Q	Sect. 1	Sect. 2	Sect. 3	Reach 1	Reach 2
		A_1	A_2	A_3	S_1	S_2
	cfs	ft^2	ft^2	ft^2	ft^3	ft^3
0.2	25,400	1,150	1,400	1,250	5.1×10^6	19.7×10^6
0.4	50,800	2,350	2,350	2,050	9.4×10^6	33.6×10^6
0.6	76,200	3,250	3,100	2,950	12.7×10^6	46.0×10^6
0.8	101,600	3,950	3,750	3,700	15.4×10^6	56.4×10^6
1.0	127,000	4,650	4,400	4,400	18.1×10^6	66.5×10^6

Reach 1. (j = 1)

6. Compute values of k and m in reach 1:
From equations (B-2) and (B-3) and for the paired [Q, S_1] data in table 2, solve for m and k. Since this example uses five sets of paired [Q, S_1] data, N = 5, and n = (5 + 1)/2 = 3.

$$\sum_{i=1}^{n} \log Q_i = \log(25,400) + \log(50,800) + \log(76,200) = 13.993$$

$$\sum_{i=n+1}^{N} \log Q_i = \log(101,600) + \log(127,000) = 10.111$$

$$\sum_{i=1}^{n} \log S_{i,1} = \log(5.1 \times 10^6) + \log(9.4 \times 10^6) + \log(12.7 \times 10^6)$$

$$= 20.785$$

$$\sum_{i=n+1}^{N} \log S_{i,1} = \log(15.4 \times 10^6) + \log(18.1 \times 10^6) = 14.445$$

Then,

$$m = \frac{(5 - 3)(13.993) - 3(10.111)}{(5 - 3)(20.785) - 3(14.445)} = 1.33$$

$$k = \log^{-1}\left\{\frac{\sum_{i=1}^{n} \log(Q_i) - m \sum_{i=1}^{n} \log(S_{i,1})}{n}\right\}$$

$$k = \log^{-1}\left\{\frac{13.993 - 1.33(20.785)}{3}\right\} = 2.82 \times 10^{-5}$$

7. Compute k^*:

From equation (16A) and for $Q_I = Q_{max} = 127,000$ cfs,

$$k^* = \frac{Q_I}{k\ V_I^m} = \frac{127,000}{2.82 \times 10^{-5}[(2000 + 1000)(43,560)]^{1.33}} = 0.072$$

8. Determine Q^* and t_o^*:

From Figure 6 and [$m = 1.33$, $k^* = 0.072$],

$$Q^* = 0.89, \qquad t_o^* = 0.22$$

9. Compute the maximum flood discharge at section 2:

From equation (14) and for [$Q_I = 127,000$ cfs, $Q^* = 0.89$]

$$Q_o = 127,000 \times 0.89 = 113,000 \text{ cfs}$$

10. Determine maximum depth of flooding at section 2:

From figure 3 and for $Q_o = 113,000$ cfs, $d_o = 16.3$ feet.

11. Compute time of maximum depth of flooding at section 2:

$$t_o = t_o^* \frac{V_I}{Q_I} = 0.22 \left(\frac{3000 x 43,560}{127,000}\right) = 230 \text{ sec} = 3.8 \text{ min}$$

Reach 2. (j = 2)

6. Compute values of k and m for reach 2:
   From equations (B-2) and (B-3) and for the [Q, S_2] data in table 2, solve for m and k.

   $m = 1.34$
   $k = 4.18 \times 10^{-6}$

7. Compute k*:
   From equation (16A) and for $Q_I = 127,000$ cfs,

$$k^* = \frac{Q_I}{k V_I^m} = \frac{127,000}{4.18 \times 10^{-6} [131 \times 10^6]^{1.34}} = 0.403$$

8. Determine Q^* and t_o^*:

   From figure 6 and [m = 1.34, $k^* = 0.403$],

   $Q^* = 0.719$, $t_o^* = 0.63$

9. Compute maximum flood discharge at section 3:
   From equation (14) and for [$Q_I = 127,000$ cfs, $Q^* = 0.719$],

   $Q_o = 127,000 \times 0.719 = 91,300$ cfs

10. Determine maximum depth of flooding at section 3:
    From figure 3 and for $Q_o = 91,300$ cfs, $d_o = 14.2$ feet.

11. Compute time of maximum depth at section 3:

$$t_o = t_o^* \frac{V_I}{Q_I} = 0.63 \frac{131 \times 10^6}{127,000} = 650 \text{ sec} = 10.8 \text{ min}$$

EXAMPLE 3

Given:

Q_{max} = 200,000 cfs

Reservoir storage volume is 8000 ac-ft

No storm inflow

The shape of the breach hydrograph is curvilinear

The discharge versus valley storage relationship in the valley reach between the dam and section 5 is represented by the equation:

$$Q = 0.976 \times 10^{-16} S^{2.5}$$

i.e., $k = 0.976 \times 10^{-16}$ and $m = 2.5$

Determine:

The maximum discharge at section 5.

Solution:

The values of k and m are given so the solution starts with procedure step 7.

7. Compute k*:

   From equation (16A)

$$k^* = \frac{Q_I}{k\,V_I^m} = \frac{200{,}000}{0.976 \times 10^{-16}\,[8000(43{,}560)]^{2.5}} = 0.904$$

8. Determine Q^*:

   From figure 4 and [$m = 2.5$, $k^* = 0.904$],

   $Q^* = 0.36$

9. Compute the maximum flood discharge at section 5:

   From equation (14) and for [Q_I = 200,000 cfs, $Q^* = 0.36$],

$$Q_o = 200{,}000 \times 0.36 = 72{,}000 \text{ cfs}$$

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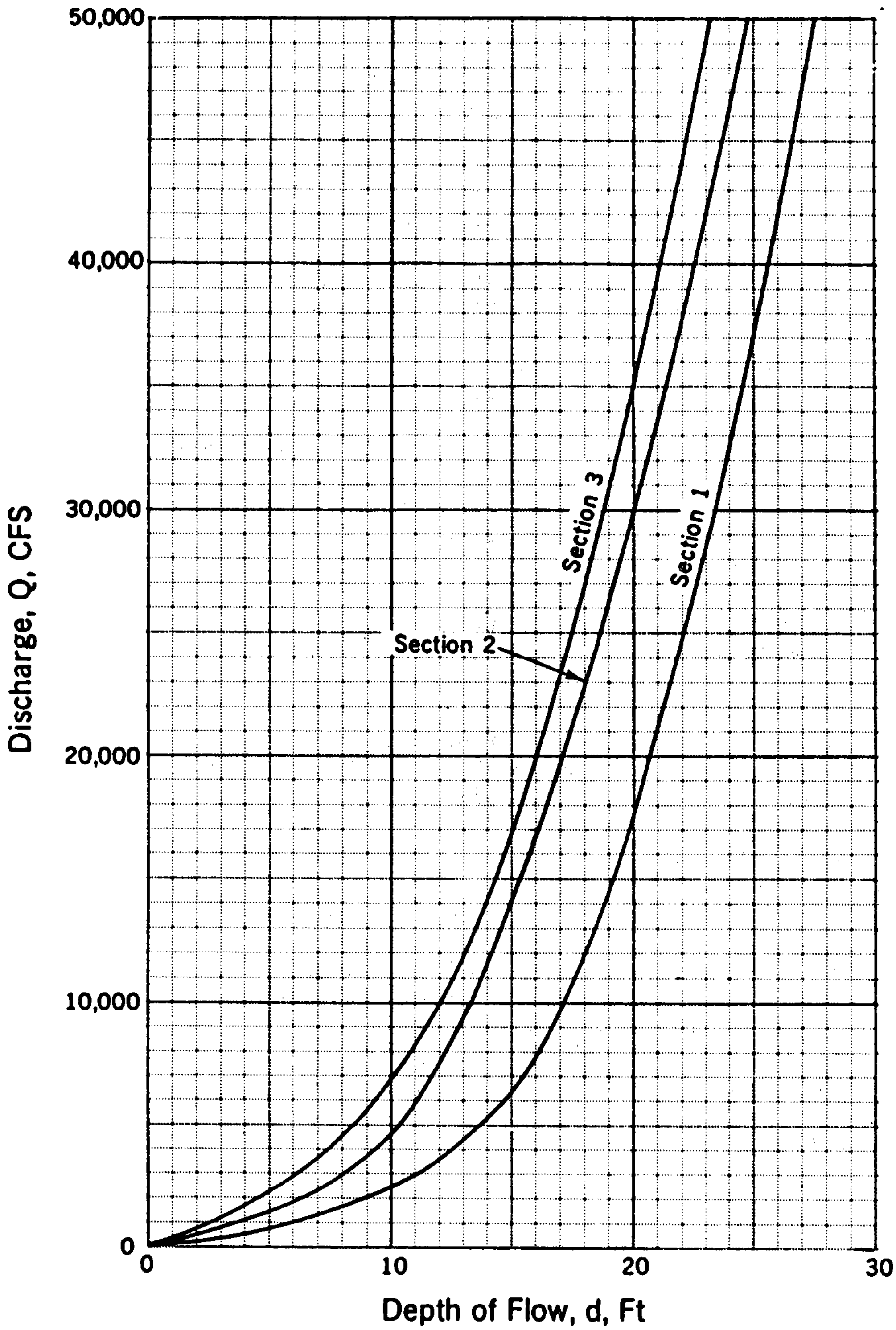


Figure 1.-- Depth-discharge curves (example 1).

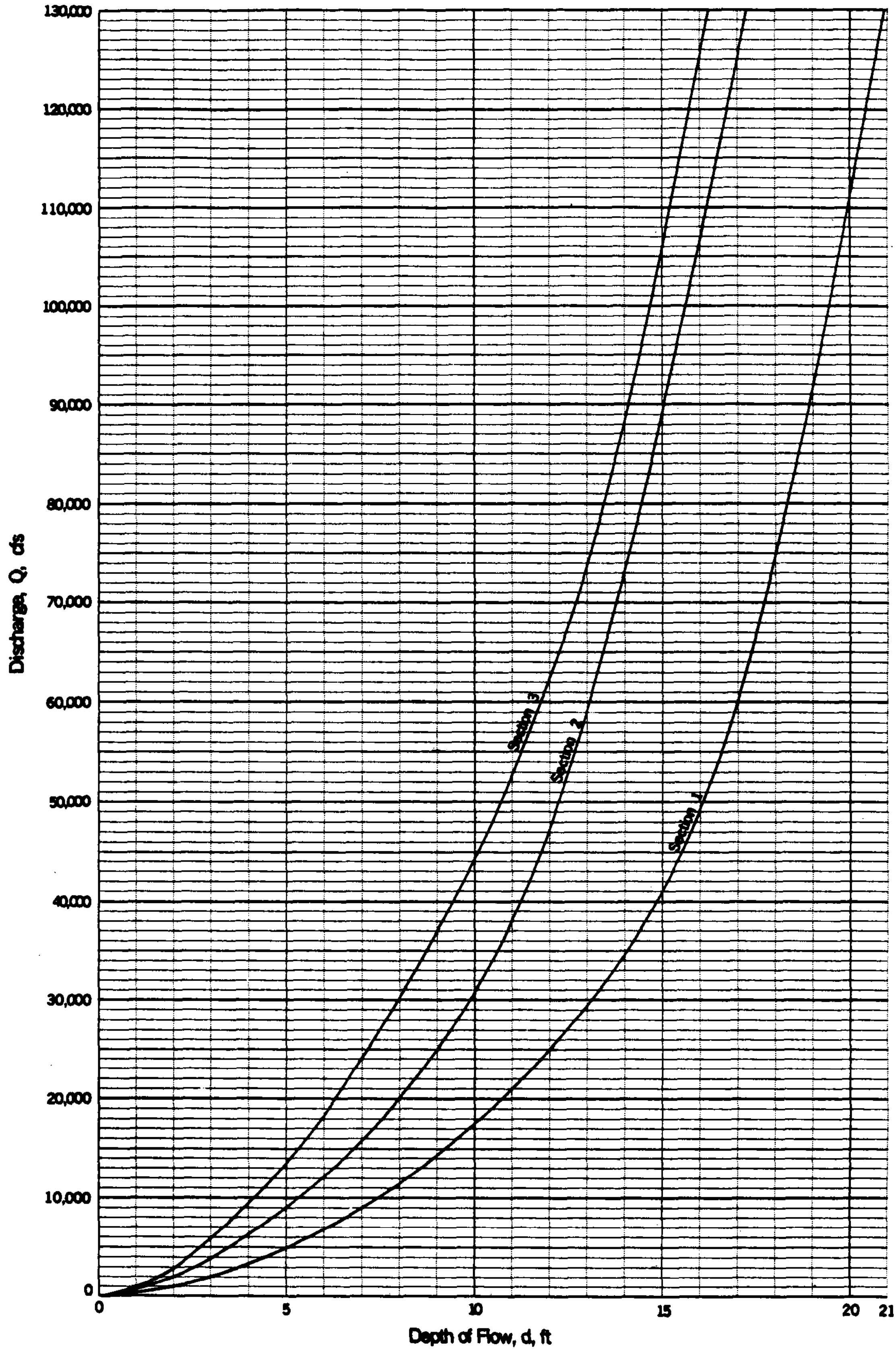


Figure 3.-- Depth-discharge curves (example 2).

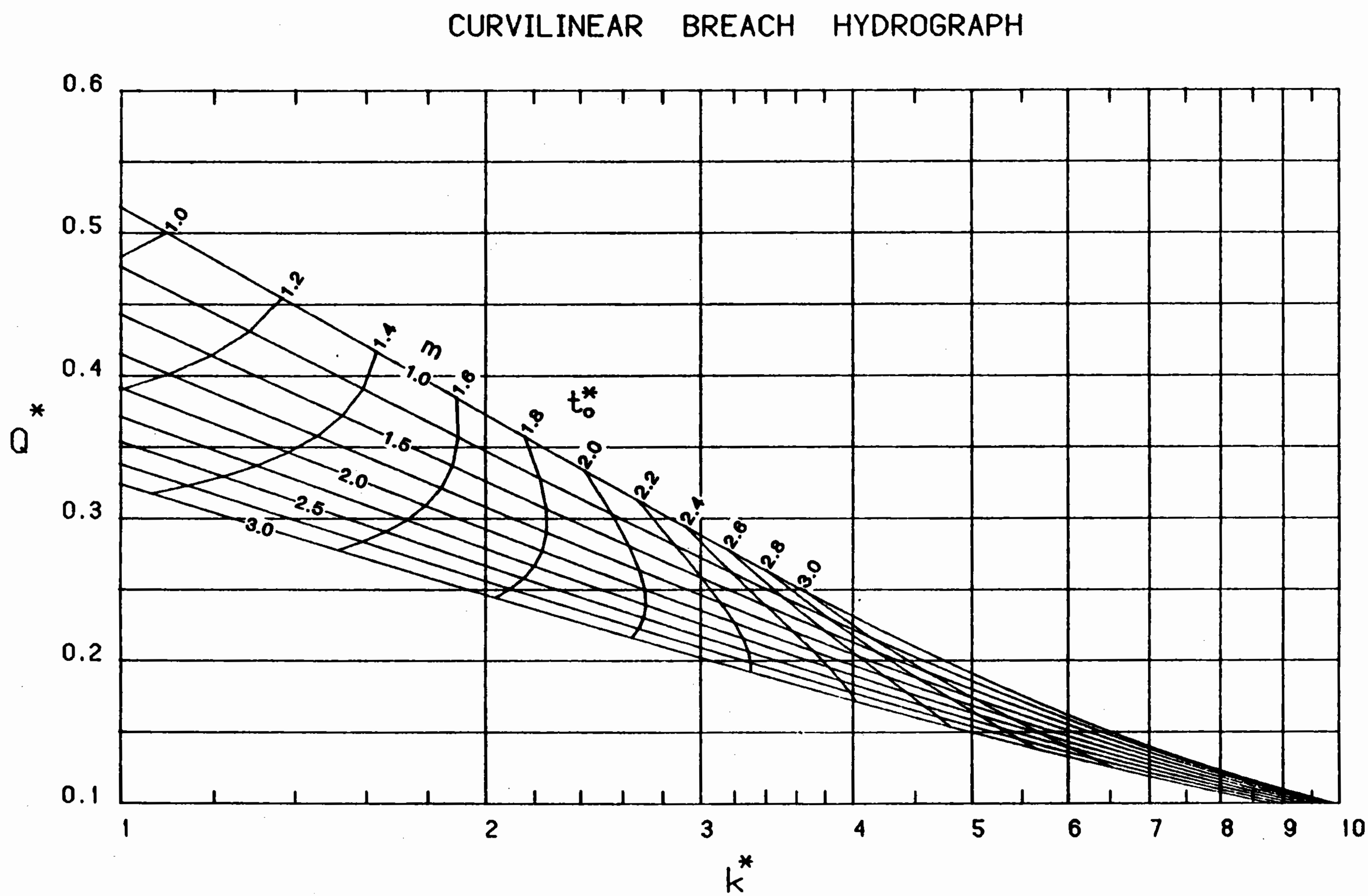


Figure 5.-- Graphical solution of simplified Att-Kin model for $1 < m < 3$ and $1 < k^* < 10$.

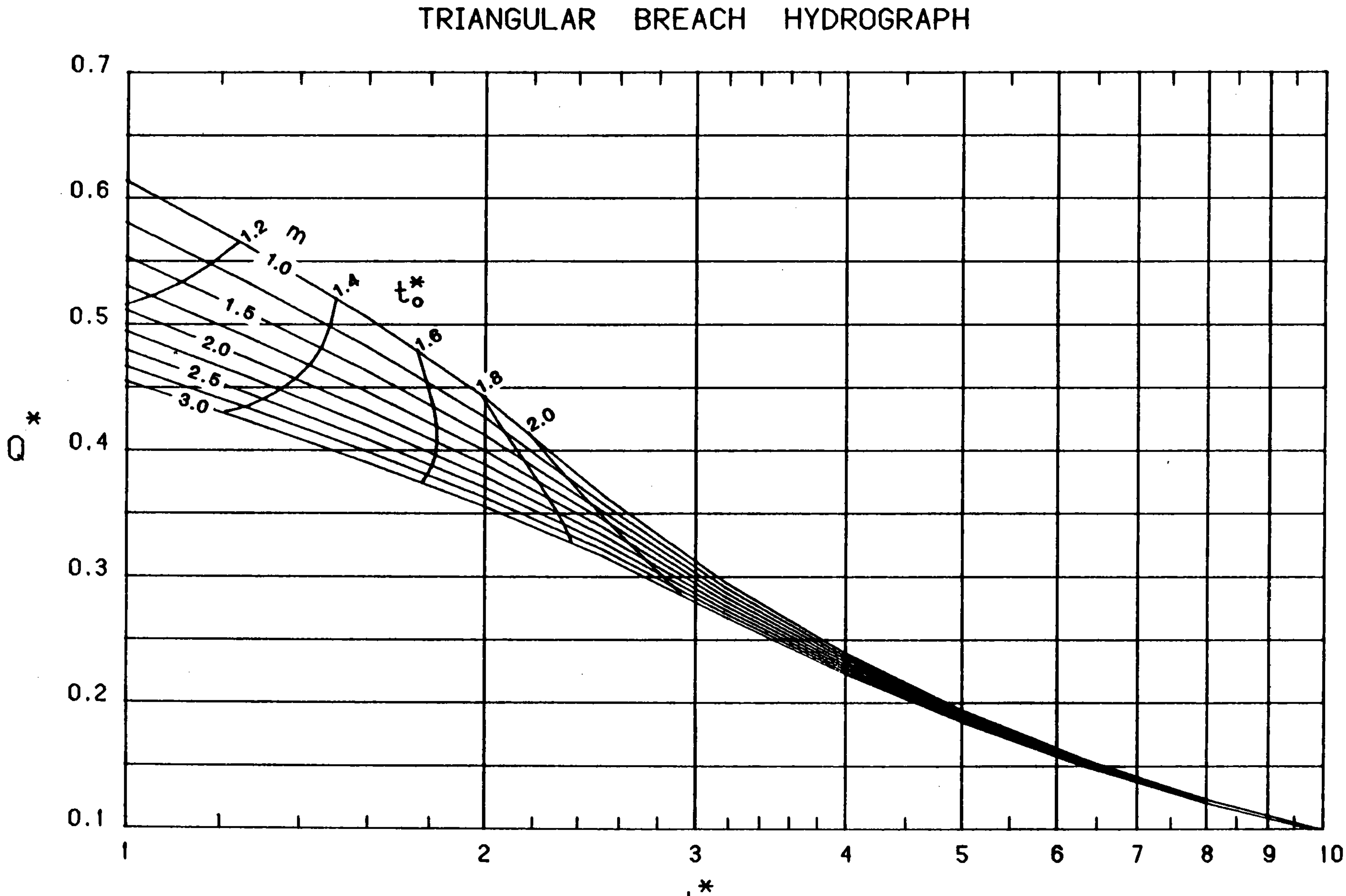


Figure 7.-- Graphical solution of simplified Att-Kin model for $1 < m < 3$ and $1 < k^* < 10$.

APPENDIX A

THE ATT-KIN FLOOD ROUTING METHOD

Introduction

From the hydraulics point of view, flood routing is a special case of unsteady flow in open channels. In most instances, the flow is treated as one-dimensional which implies that spatial variation of physical quantities such as discharges, velocities, and flow depths is significant only along the direction of flow. Consequently, the mathematical model simulating prototype behavior consists only of two partial differential equations; namely, the continuity equation describing the principle of conservation of mass and the equation of motion describing the principle of conservation of momentum in the direction of the flow.

In the general case of nonprismatic channels, the two equations known as the Saint-Venant equations assume the form:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{(A-1)}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + A\, g\, \frac{\partial d}{\partial x} = A\, g\, (S_g - S_f) \qquad \text{(A-2)}$$

in which,

$Q \equiv$ discharge at point $\{x,t\}$ on the x, t plane
$A \equiv$ valley cross-section at the same point
$d \equiv$ flow depth associated with Q and A
$S_g \equiv$ longitudinal gradient of valley floor
$S_f \equiv$ friction gradient at point $\{x,t\}$

Added to the above two are the three auxiliary relationships:

$$d \equiv f_1(A) \qquad (A-3)$$

$$r \equiv f_2(A) \qquad (A-4)$$

$$S_f \equiv f_3(Q,A,r,n) \qquad (A-5)$$

in which,

$r \equiv$ hydraulic radius of the flow section

$n \equiv$ roughness coefficient in Manning's equation

Relationships (A-3) and (A-4) are developed from geometric data included in the input vector. The appearance of Manning's n in the right-hand side of equation (A-5) does not mandate exclusive use of Manning's formula; any reputable friction formula is acceptable.

The simultaneous solution of the above two partial differential equations together with the three auxiliary ones produces paired values of the two dependent variables Q and A as discrete functions of the independent variables x and t, as well as, of the predefined fixed variables S_f and n. In mathematical jargon, the solution describes the response of the system to the outside stimuli and constraints stipulated by the boundary conditions.

HYDRAULIC FLOOD ROUTING

The definition of the term refers to situations satisfying the following requirements with regard to boundary conditions and method of solution:

There are at least two independent boundary conditions; one of them is the flood hydrograph applied at the upstream end of the simple linear channel system considered here, and the other defines the flood flow area A as a function of time or discharge Q. The second boundary condition applies at the upstream end valley cross-section, if the flow is supercritical, and at the downstream end section of the system when the flow is subcritical. Equation (A-2) may be simplified through the elimination of negligible terms, but it has to be solved simultaneously with equation (A-1). The solution of the mathematical model is, in other words, "coupled."

The distinct characteristic of hydraulic routing is reflected in the associated problem statement which is as follows:

> Given the state of the system at time t, i.e., all fixed or dependent variables defined along the entire length of the channel, determine the state of the system for time $t + \Delta t$.

Because of the way the solution proceeds, it is said to be "marching forward with time." In the process, the solution produces at the end of each time interval instantaneous profiles of discharge and flow area or of corresponding depth of flow, flow stage, and water surface elevation along the entire length of the reach. The supporting data is stored and used at the completion of computations to construct routed hydrographs and histograms of stage, flow area, or water surface elevation at any internal or boundary valley cross-section. It is evident that the prerequisite for starting the solution is the specification of two initial conditions; that is, profiles of stage or water surface elevation and discharge at time zero along the entire length of the reach.

HYDROLOGIC FLOOD ROUTING

The preceding brief outline of concepts underlying hydraulic flood routing can serve as the framework for comparing and discussing the class of hydrologic flood routing methods, of which the Att-Kin method is a member.

The characteristic features of the methods in the hydrologic class are:

1. The first boundary condition is the same as for the hydraulic class. But, while previously, the application of the predefined single-valued relationship between Q and A was restricted only to the end cross-

sections; in the hydrologic class, it is completely unrestricted. Thus, there are as many such relationships as there are valley cross-sections.

2. The solution starts at the upstream end of the channel with the inflow hydrograph as the boundary condition and advances frontally in the direction of the flow. Thus, in contrast to the hydraulic routing scheme, the solution "marches forward with distance." In the conventional procedure, the total valley reach is divided, arbitrarily or as dictated by the pertinent input data, into a number of subreaches. Each subreach, beginning with the most upstream, is dealt with completely before proceeding with the solution of the flood routing problem in the subreach immediately downstream. The mathematical model is solved repeatedly at predefined time intervals for each subreach, generating the necessary data for the complete definition of the outflow hydrograph from the subreach. This hydrograph is used at the beginning of the next computational cycle as the inflow hydrograph to the subreach immediately downstream.

The problem statement of hydrologic flood routing methods is:

Given the complete hydrograph at valley section j, construct the hydrograph at section j+1.

3. The continuity equation (A-1) may be used as is or in the integral form,

$$S_2 = S_1 + (\bar{Q}_2 - \bar{Q}_1)\,\Delta t \qquad \text{(A-6)}$$

in which,

$S_1 \equiv$ valley storage at time t_1

$S_2 \equiv$ valley storage at time $t_2 = t_1 + \Delta t$

$\bar{Q}_1 \equiv$ average outflow from the reach during the time interval $\Delta t = t_2 - t_1$

$\bar{Q}_2 \equiv$ average inflow into the reach during the time interval $\Delta t = t_2 - t_1$

4. The equation of motion (A-2) is simplified through elimination of the first term, the first and the second, or all three terms on the left-hand side of the equation. Elimination of the first term is mandatory and of the other two, optional. The simplification not only reduces the

complexity of the partial differential equation in time and space but also transforms it into an ordinary differential equation in space only. Then, the simplified equation can be integrated along the direction of flow independently of the continuity equation (A-6). Integrating of the equation along the direction of flow allows for the development of the discharge versus flow area relationships through conventional steady-state water surface profile techniques.

So, in brief, simplification of the equation of motion (A-2) is allowed in all methods, regardless of class. Yet, whereas for methods in the hydraulic class, the elimination of the term $\partial Q/\partial t$ is optional; for those in the hydrologic class, it is mandatory. Integration of the simplified equation (A-2) is mandatory, whereas, for those in the hydraulic class, it is forbidden. Because of the way the associated mathematical models are integrated, the solutions of the hydrologic methods are called "uncoupled."

5. Results from independent integration of the reduced equation (A-2), which, in essence, are data generated from steady-state water surface profile computations, are used next to develop single-valued relationships between discharge and associated valley-storage or associated local hydraulic parameters, such as valley cross-section, stage, and water surface elevation.

Typical discharge-storage and discharge-flow area relationships are:

$$Q = k\, S^m \qquad \text{(A-7)}$$

$$Q = k_o\, A^m \qquad \text{(A-8)}$$

Hydrologic flood routing models are formed by combining equation (A-1) or equation (A-6) with either equation (A-7) or (A-8).

6. Generally, no attempt is made by hydrologic flood routing methods, although the necessary data is available, to test whether or not the instantaneous water surface profiles produced by the solution satisfy the second equation of their model.

7. The built-in stability of the hydrologic flood routing models renders their solutions immune to hazards created by extraordinary boundary or initial conditions that often prove fatal to hydraulic routing models. Specifically, most hydrologic models are capable of admitting inflow hydrographs exhibiting instantaneous rises and of handling initial conditions of any type, including the dry-bed one.

HYDROLOGIC FLOOD ROUTING METHODS

Hydrologic routing methods can be identified further by subclass according to formulation of their mathematical model. The two most distinct groups, in that respect, are the subclasses of "storage" routing methods and of "kinematic" routing methods. Salient features of the two groups are discussed below.

"Storage" flood routing methods are hydrologic flood routing methods in whose mathematical models continuity is represented by equation (A-6) and the equation of motion is represented by equation (A-7).

A typical formulation of mathematical models in the subclass is,

$$S_2 = S_1 + (\overline{Q}_2 - \overline{Q}_1)\,\Delta t \qquad \text{(A-6)}$$

$$Q = k_s\, S^{m_s} \qquad \text{(A-7A)}$$

and since

$$\overline{Q}_1 = \frac{Q_{1,1} + Q_{1,2}}{2}$$

$$S_2 + \frac{Q_{1,2}}{2}\,\Delta t = S_1 + \left(\overline{Q}_2 - \frac{Q_{1,1}}{2}\right)\Delta t \qquad \text{(A-9)}$$

Equation (A-9) is solved simultaneously with equation (A-7A). Solving the system of the two equations can be done numerically using a tabular form or semi-graphically using reference graphs.

The postulate in most storage flood routing methods is that the coefficient and the exponent in equation (A-7A) are the same as those in equation

(2). Consequently, their values can be determined by methods similar to the one described in Appendix B.

"Kinematic" routing methods are hydrologic flood routing methods whose models express continuity by equation (A-1) and dynamic equilibrium by equation (A-8).

The typical mathematical model of the subclass is,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{(A-1)}$$

$$Q = k_o A^m \qquad \text{(A-8)}$$

Application of the chain differentiation rule on equation (A-1), yields,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial Q}\frac{\partial Q}{\partial t} = 0 \qquad \text{(A-10)}$$

Also, by setting the total differential of Q equal to zero, we get

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt = 0 \qquad \text{(A-11)}$$

Eliminating the terms $\frac{\partial Q}{\partial x}$ and $\frac{\partial Q}{\partial t}$ between equations (A-10) and (A-11), and completing the algebra results in,

$$\frac{dx}{dt} = \frac{\partial Q}{\partial A} \qquad \text{(A-12)}$$

The mathematical interpretation of equations (A-11) and (A-12) is that the value of the characteristic function Q remains constant along the characteristic line whose differential equation on the x, t plane is equation (A-12). The corresponding hydraulic interpretation is that during the time interval Δt the instantaneous discharge through any channel section will be translated without change to another section located at a distance Δx downstream, such that,

$$\Delta x = \frac{dx}{dt} \Delta t = \frac{\partial Q}{\partial A} \Delta t$$

From which, the speed of travel of discharge Q is defined as

$$c = \frac{dx}{dt} = \frac{\partial Q}{\partial A}$$

The quasi-steady formulation resulting in equation (A-8) allows substituting total derivatives for partial ones.

Hence,

$$c = \frac{\partial Q}{\partial A} = \frac{dQ}{dA} = m \; k \; A^{m-1} = m \frac{Q}{A} = m \; V \qquad \text{(A-13)}$$

V ≡ average flow velocity through cross-section j at the beginning of time interval Δt.

The problem statement in kinematic flood routing is:

Given: The profile of discharge in the reach between cross-sections 2 and 1 at time t_1, and the relationship,

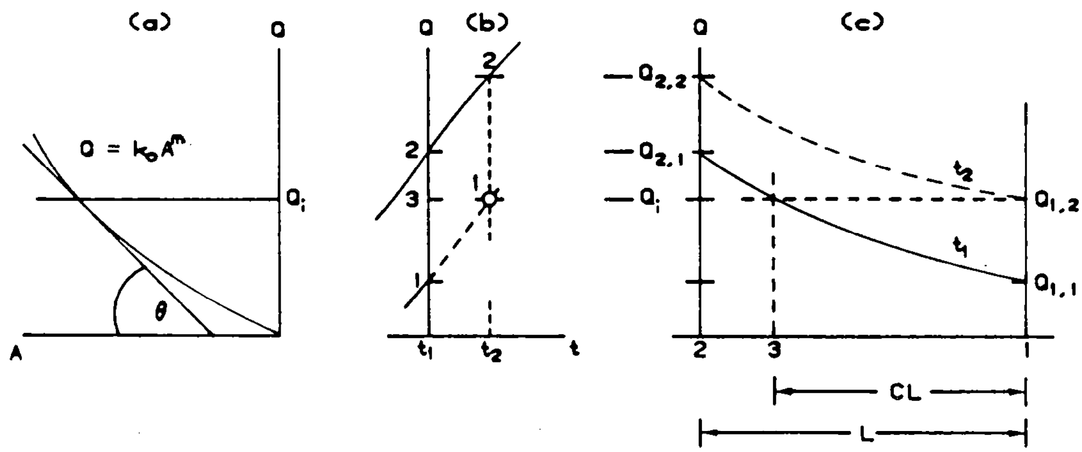
$$Q = k_o A^m$$

Find: The outflow discharge $Q_{1,2}$ from the reach at time t_2.

The concept underlying the kinematic approach is that at time t_1, there is a discharge Q_i through section 3, located between sections 1 and 2, moving downstream at a speed of $dQ/dA = \tan \Theta$. Then, during the time interval $t_2 - t_1$, the discharge Q_i will travel the distance between sections 3 and 1 to become the outflow discharge $Q_{1,2}$ at time t_2. The sketches below illustrate the concept.

The curve in sketch (a) represents the relationship between the discharge and flow area at section 3 in the reach, located at distance CxL, $0 < C < 1$, upstream from section 1.

Sketch (b) shows segments of the inflow and outflow hydrographs, i.e., at sections 2 and 1, respectively. The former is defined for all times and the latter to time t_1.



Finally, the curve on sketch (c) simulates the assumed known discharge profile in the reach at time t_1.

So, the only thing one has to do is,

assume a value of C

compute the distance CxL

find from the profile on sketch (c) the associated discharge

determine the angle θ from sketch (a)

set $c = \tan\theta$, and check whether or not $c = \frac{CxL}{t_2 - t_1}$

The main difficulty in practice is with defining adequately the curve on sketch (c) at time t_1, and to a lesser degree, the curve on sketch (a).

THE ATT-KIN METHOD

The Att-Kin method is a hydrologic flood routing method differing from the norm in the following two major aspects:

In contrast to the conventional arrangements of valley subreaches in series, the subreaches in the Att-Kin method are arranged, so to speak, in parallel. In other words, each subreach begins at the first valley cross-section upstream and ends at some other cross-section downstream. So, the same flood hydrograph is routed independently through each subreach and, if so desired, in arbitrary order.

In contrast to the conventional treatment of the storage and kinematic flood routing submodels as mutually exclusive, the Att-Kin method, on the strength of the observation that neither submodel is able to account fully for the required valley storage, uses both submodels. Specifically, it uses the storage routing submodel to determine the part of valley storage in the subreach required for attenuation of the peak and the kinematic submodel to determine the part of the valley storage associated with the transformation imposed upon the outflow hydrograph $Q_1(t)$ by the kinematic nature of the flood wave.

THE ATT-KIN MATHEMATICAL MODEL

The storage submodel consists of the conventional two equations mentioned in page A-6. These are:

$$Q = k_s \, S^{m_s} \qquad \text{(A-7A)}$$

$$S_2 + \frac{Q_{1,2}}{2} \Delta t = S_1 + (\bar{Q}_2 - \frac{Q_{1,1}}{2}) \, \Delta t \qquad \text{(A-9)}$$

Solution of the submodel for a given inflow hydrograph produces the following data:

$Q_o \equiv$ peak outflow

$V_s \equiv$ associated valley storage

$t_s \equiv$ time of occurrence of Q_o

and

$Q_1(t) \equiv$ storage routed hydrograph

The transformation of the outflow hydrograph $Q_1(t)$ consists of a relative distortion and a pure translation, both with respect to time. The associated volumes are determined through application of the concept of kinematic routing described in item "Kinematic" routing methods, pages A-7 and A-8.

The distortion of the hydrograph is caused by the fact that large discharges travel at a different speed than smaller ones, as indicated by equations (A-8) and (A-9). Consequently, the time required for the former to travel the distance L, the length of the reach, will be different than for the latter.

According to equation (A-13), the speed of travel of a discharge $Q = Q_1(t)$ is $c = \frac{dQ}{dA}$. In nonuniform flows, the speed $\frac{dQ}{dA}$ is meaningless when the symbol A denotes the flow area associated with Q at a particular cross-section of the subreach, as implied by equation (A-8). The logical area, in this case, is the mean flow area in the subreach, defined as,

$$\bar{A} = \frac{1}{L} \int_0^L A dx \qquad \text{(A-14)}$$

Then, equation (A-13) becomes,

$$c = \frac{dQ}{d\bar{A}} = \frac{dQ}{dS} \frac{dS}{d\bar{A}} \qquad \text{(A-14A)}$$

and since, by definition,

$$S = \int_0^L A dx \qquad \text{(A-15)}$$

eliminating the integrals on the right-hand sides of equations (A-14) and (A-15) results in $S = \bar{A} L$, from which

$$\frac{dS}{d\bar{A}} = L$$

and $$c = \frac{dQ}{dS} \frac{dS}{d\bar{A}} = L \frac{dQ}{dS} \qquad \text{(A-16)}$$

Associated with the speed of travel c of the discharge Q is a time of travel t_1, such that

$$c = \frac{L}{t_1}$$

from which

$$t_1 = \frac{L}{c} = \frac{dS}{dQ} \qquad \text{(A-17)}$$

For the peak discharge Q_o

$$t_{1,o} = \frac{dS_o}{dQ_o}$$

so, by definition,

$$\delta t = t_1 - t_{1,o} = \frac{dS}{dQ} - \frac{dS_o}{dQ_o} \qquad \text{(A-18)}$$

$\delta t \equiv$ kinematic distortion of the outflow hydrograph associated with the discharge $Q = Q_1(t)$ relative to the distortion of the peak outflow discharge Q_o.

The variables Q_o and S_o are related to each other through equation (A-7) which solved for S_o becomes,

$$S_o = \left(\frac{Q_o}{k}\right)^{\frac{1}{m}} \qquad \text{(A-18A)}$$

$S_o \equiv$ valley storage in the reach associated with peak outflow Q_o.

The associated infinitesimal volume resulting from the distortion of an increment of discharge dQ is,

$$dV_d = \delta t \ dQ = \left[\frac{dS}{dQ} - \frac{dS_o}{dQ_o}\right] dQ$$

from which

$$V_d = \int_o^{Q_o}\left[\frac{dS}{dQ} - \frac{dS_o}{dQ_o}\right] dQ \qquad \text{(A-19)}$$

$V_d \equiv$ valley storage attributed to kinematic distortion.

Integration of the right-hand side of equation (A-19) by parts gives,

$$\int_o^{Q_o} \frac{dS}{dQ}\, dQ = \int_o^{S_o} dS = S_o$$

and from equation (A-18A),

$$\int_o^{Q_o} \frac{dS_o}{dQ_o}\, dQ = \frac{dS_o}{dQ_o}\int_o^{Q_o} dQ = \frac{dS_o}{dQ_o}\, Q_o = \frac{S_o}{m}$$

Hence,

$$V_d = S_o\left(1 - \frac{1}{m}\right) \qquad \text{(A-20)}$$

The pure translation of the outflow hydrograph is equal, by definition, to the translation of the peak outflow Q_o. It may be explained, physically, as the time lapse between the occurrence of the peak inflow at the upstream end of the reach and the occurrence of the peak outflow at the downstream end. Then, according to the kinematic concept,

$$t_o - t_I = \frac{L}{\frac{dQ}{d\bar{A}}} = \frac{\frac{dS}{d\bar{A}}}{\frac{dQ}{d\bar{A}}} = \frac{dS}{dQ}$$

in which

$t_o \equiv$ time to peak outflow Q_o
$t_I \equiv$ time to peak inflow Q_I

In the expression above, Q varies between Q_o and Q_I and S between S_o and S_I. For the evaluation of the term dS/dQ, the Att-Kin method uses the approximation below,

$$\frac{dS}{dQ} \simeq \frac{S_I - S_o}{Q_I - Q_o}$$

which is considered adequate for the purpose. Thus, the expression for the time to peak outflow becomes,

$$t_o \simeq t_I + \frac{S_I - S_o}{Q_I - Q_o} \qquad \text{(A-21)}$$

$S_I \equiv$ valley storage associated with Q_I from equation (A-7).

Then,

$$V_i = \int_0^{t_o} Q_2(t)dt = \int_0^{t_s} Q_2(t)dt + \int_{t_s}^{t_o} Q_2(t)dt$$

$V_i \equiv$ total volume of inflow into the reach from time zero to time t_o.

The volume represented by the first integral on the right-hand side of the equation above is equal to the sum of three volumes; the valley storage associated with Q_o and determined by the storage routing sub-model, the part of valley storage attributed to the distortion of the outflow hydrograph, and the net volume of outflow from the reach during the time interval between zero and t_o. Thus,

$$\int_{o}^{t_s} Q_2(t)dt = V_s + V_d + V_o$$

$V_o \equiv$ net volume of outflow from the reach to time t_o.

Letting,

$$V_t = \int_{t_s}^{t_o} Q_2(t)dt \qquad \text{(A-22)}$$

$$V_i = V_s + V_d + V_o + V_t$$

Hence,

$$S_n = V_i - V_o = V_s + V_d + V_t \qquad \text{(A-22A)}$$

$S_n \equiv$ net valley storage in the reach at time t_o

The key argument in the Att-Kin method is that the net storage as determined by equation (A-22A) is equal to the valley storage computed from equation (A-18A). In other words,

$$S_o = S_n = V_s + V_d + V_t$$

In summary, the mathematical model of the Att-Kin method consists of the following ten equations:

$$S_{1,i} + \frac{Q_{1,i}}{2} \Delta t = S_{1,i-1} + (\bar{Q}_{2,i} - \frac{Q_{1,i-1}}{2}) \Delta t \qquad \text{(A-23)}$$

$$Q_1(t) = k_s S^{m_s} \qquad \text{(A-24)}$$

$$Q_o = Q_1(t_s) = k_s V_s^{m_s} \qquad \text{(A-25)}$$

$$t_s = \sum \Delta t \qquad \text{(A-26)}$$

$$Q_o = k S_o^m \qquad \text{(A-27)}$$

$$Q_I = k S_I^m \qquad \text{(A-28)}$$

$$V_d = S_o(1 - \frac{1}{m}) \qquad \text{(A-29)}$$

$$t_o = t_I + \frac{S_I - S_o}{Q_I - Q_o} \qquad \text{(A-30)}$$

$$V_t = \int_{t_s}^{t_o} Q_2(t)dt \qquad \text{(A-31)}$$

$$S_o = V_s + V_d + V_t \qquad \text{(A-32)}$$

The variables and their number in the above equations are:

Equation		
(A-23)	S_1, Q_1, Δt, Q_2	4
(A-24)	k_s, m_s	2
(A-25)	Q_o, t_s, V_s	3
(A-27)	k, m, S_o	3
(A-28)	Q_I, S_I	2
(A-29)	V_d	1
(A-30)	t_o, t_I	2
(A-31)	V_t	1
	for a total number of	18

The predetermined variables are the following six:

Q_2, Q_I, t_I, Δt, k, and m.

It appears that the system of the Att-Kin mathematical model consisting of ten independent equations containing eighteen variables, of which six variables are treated as independent, is two equations short of consistent. Actually, the system is only one equation short because of the relationship between k and k_s resulting from elimination of Q_o between equations (A-25) and (A-27). The problem of the missing equation is resolved in the Att-Kin method by assuming

$$m_s = m$$

Then, the relationship between k and k_s mentioned earlier becomes,

$$k_s = k \left(\frac{S_o}{V_s}\right)^m$$

Consequently, equation (A-24) becomes,

$$Q_1 = k \left(\frac{S_o}{V_s}\right)^m S^m \qquad \text{(A-33)}$$

Equations (A-23) and (A-33) are the components of the storage routing submodel used by the Att-Kin method.

PROBLEM STATEMENT AND SOLUTION PROCEDURE

Given the input data set:

Inflow hydrograph data $\{ t, Q_2(t) \}$, Q_I, t_I

the computational time interval Δt

and the values of k and m,

Find:

(a). The peak outflow Q_o and the time to peak t_o.

(b). A set of paired values [Q_1, t] of the outflow hydrograph.

Solution Procedure:

(a). Setting,

$$C = \frac{V_s}{S_o} \quad , 0 < C < 1$$

equation (A-33) becomes,

$$Q_1 = k \left(\frac{S}{C} \right)^m \qquad \text{(A-34)}$$

Because of its structure, the Att-Kin mathematical model does not afford a direct solution. The procedure for solving the model indirectly by means of an iterative technique about the value of the coefficient C is given below.

1. Assume a value of C. Due to the constraint imposed by equations (A-29) and (A-32), the selected value of C must satisfy the condition,

$$(V_s + V_d) \leq S_o$$

or

$$(1 - \frac{1}{m} + C) \leq 1$$

Thus, the selected value of C must be equal to or smaller than $\frac{1}{m}$.

In the event C = 1/m, $S_o = V_s + V_d$. Then, from equation (A-32), $V_t = 0$, and, from equation (A-31), $t_o = t_s$. The physical interpretation of $t_o = t_s$ is that the storage routed flood hydrograph will be attenuated and distorted with time, but not translated.

2. Perform storage routing computations using equations (A-23) and (A-34) to determine Q_o, V_s, t_s, and the array $\{ Q_1, t \}$.

3. Compute S_o from equation (A-27) solved for S_o, using given values of k and m, and the value of Q_o from step 2.

4. Compute V_d from equation (A-29) using given value of m and the value of S_o from step 3.

5. Compute S_I from equation (A-28) solved for S_I, using input values of m, k, and Q_I.

6. Compute t_o from equation (A-30) using input values of t_I and Q_I, and known values of Q_o, S_o, and S_I from steps 2, 3, and 5, respectively.

7. Compute V_t from equation (A-31) using input values of $Q_2(t)$ and known values of t_s and t_o from steps 2 and 6, respectively.

8. Compute S_o from equation (A-32) using known values of V_s, V_d, and V_t, from steps 2, 4, and 7, respectively.

9. If value of S_o from step 8 is close to the value of S_o from step 3, go to step 11.

10. Modify C, enter the new value in equation (A-34), and return to step 2.

11. The sought for values of Q_o and t_o are those from steps 2 and 6, respectively.

(b).

The procedure below determines the displacement with time of a point on the storage routed flood hydrograph whose coordinates on the Q, t plane determined in part (a) of the solution are $Q_{1,i}$, $t_{1,i}$.

This part of the solution procedure uses the basic relationship between valley storage in the reach and outflow discharge represented by equation (A-7) solved for S. i.e.,

$$S = \left(\frac{Q}{k}\right)^{\frac{1}{m}} \qquad \text{(A-35)}$$

Through the substitution of $S_{1,i}$ for S, and of $Q_{1,i}$ for Q, equation (A-35) becomes,

$$S_{1,i} = \left(\frac{Q_{1,i}}{k}\right)^{\frac{1}{m}}$$

Hence,

$$\frac{dS_{1,i}}{dQ_{1,i}} = \frac{1}{m\ k^{1/m}}\ Q_{1,i}^{\frac{1}{m} - 1}$$

Similarly, using equation (A-27) and the parameters S_o and Q_o, results in,

$$\frac{dS_o}{dQ_o} = \frac{1}{m\ k^{1/m}}\ Q_o^{\frac{1}{m} - 1}$$

1. Use input values of k and m, the value of Q_o determined in part (a), and the selected outflow discharge $Q_{1,i}$ in equation (A-18), to compute,

$$\delta t = \frac{dS_{1,i}}{dQ_{1,i}} - \frac{dS_o}{dQ_o} = \frac{1}{m\ k^{1/m}}\left(Q_{1,i}^{\frac{1}{m} - 1} - Q_o^{\frac{1}{m} - 1} \right)$$

   $\delta t \equiv$ time displacement of discharge $Q_{1,i}$ due to kinematic distortion.

2. Use the values of t_s and t_o from steps 2(a) and 6(a), respectively, in the definition below to compute δt_o

   $\delta t_o = t_o - t_s$

   $\delta t_o \equiv$ time displacement of discharge $Q_{1,i}$ due to kinematic translation.

3. Compute the total kinematic time lag of discharge $Q_{1,i}$ from,

$$\delta t_{1,i} = \delta t_i + \delta t_o$$

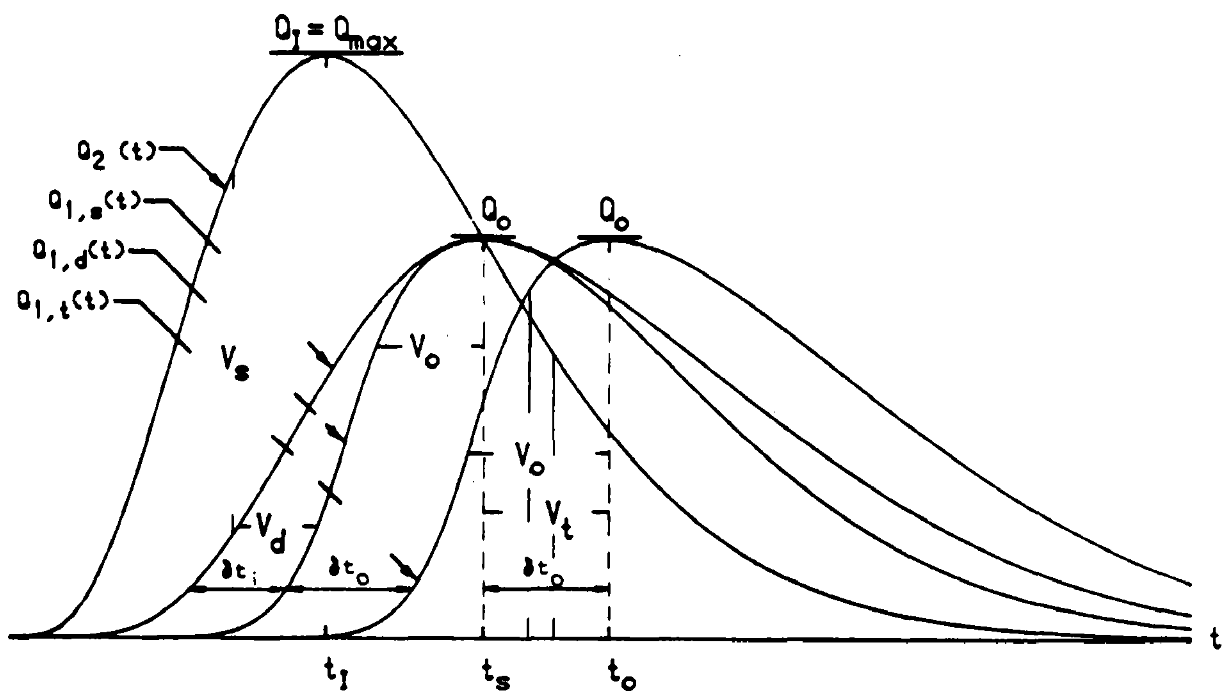
The time lag of a discharge is the same for the rising and falling limb of the storage routed hydrograph.

4. Determine the time coordinate of the Att-Kin routed discharge $Q_{1,i}$ from,

$$t_{1,i} + \delta t_{1,i}$$

The procedure is repeated as many times as needed with different values of Q, where $0 < Q < Q_o$. Since the pure translation, δt_o, is the same for all points of the hydrograph, step 2(b) may be omitted in all computational cycles after the first one.

The sketch below serves the dual purpose of defining symbols and of describing visually the unique features of flood routing by the Att-Kin method.



APPENDIX B

Determination of exponent m and coefficient k in valley reach j by the "method of averages"; (Smith and Gale 1956)

The procedure used by the method to determine m and k consists of the following steps:

1. Substitute paired Q, S_j data into equation (3), appearing below as equation (B-1), to obtain the same number of equations as there are pairs of such data.

$$\log Q_i = \log k + m \log S_{i,j} \qquad \text{(B-1)}$$

2. Divide the resulting equations into two groups, with each group having as nearly as possible the same number of equations.

3. Compute the following partial sums of the terms on the left- and right-hand sides of the equations in the two groups:

$$\sum_{i=1}^{n} \log Q_i \qquad \sum_{i=n+1}^{N} \log Q_i$$

$$\sum_{i=1}^{n} \log S_{i,j} \qquad \sum_{i=n+1}^{N} \log S_{i,j}$$

in which:

$N \equiv$ number of paired Q_i, $S_{i,j}$ data

$n \equiv \frac{N}{2}$ when N is even, and $\frac{N+1}{2}$ when N is odd

$Q_i \equiv$ outflow discharge in cfs

$S_{i,j}$ ≡ valley storage in ft_3 associated with discharge Q_i in reach j; i.e., the valley reach between sections 1 and j+1.

4. Compute the values of m and k from equations (B-2) and (B-3), respectively:

$$m = \frac{[N - n] \sum_{i=1}^{n} \log Q_i - n \sum_{i=n+1}^{N} \log Q_i}{[N - n] \sum_{i=1}^{n} \log S_{i,j} - n \sum_{i=n+1}^{N} \log S_{i,j}} \qquad \text{(B-2)}$$

$$k = \log^{-1} \left[\frac{\sum_{i=1}^{n} \log Q_i - m \sum_{i=1}^{n} \log S_{i,j}}{n}\right] \qquad \text{(B-3)}$$