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# Scale-free networks, 1/f dynamics, and nonlinear conflict size scaling from an agent-based simulation model of societal-scale bilateral conflict and cooperation



PHYSICA

STATISTICAL MECHANIC

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#### ABSTRACT

An agent-based model is presented that mechanistically simulates social interactions across two partially coupled lattices, each containing a mixture of individualists, networkers, and reciprocators. Numerical experiments reveal evidence for two spontaneously emergent and widely relevant complex behaviors: self-organized criticality generating fractal (1/f) dynamics, and a scale-free (power-law degree distribution) network, adding to the short list of generative mechanisms for these phenomena. The model may also suggest explanatory hypotheses for two sociological puzzles: Richardson's scaling law for war size; and an inverse relationship between actor scale and water resource conflict, potentially relevant to this century's prognosticated water wars. Adjusting a handful of model parameters yields a diverse set of fundamentally different behaviors, perhaps implying model applicability to a wide range of social systems and that comparatively simple social engineering steps could conceivably induce large social shifts.

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#### 1. Introduction

Computational treatments of social systems have advanced tremendously with explosive growth in social media, big data, and machine learning, contributing insights into topics ranging from cooperation and conflict in collaborative environments like Wikipedia to the surprisingly nuanced dynamics of opinion formation around climate change [1–4]. However, such data-driven results are by nature specific to the dataset studied, and there is concern that work on generative mechanisms and process explanations (see, e.g., Ref. [5] for a review), ultimately leading to process-based predictions, has lagged [1,2,6]. More broadly, this knowledge gap may extend beyond sociophysics to include complex system science as a whole, spanning social, biological, physical, and engineered systems. Indeed, such considerations have led to public debates among celebrated scientists about the nature, reach, and abilities of modern (and post-modern) science and again reinforce the value of concentrating on underlying generative process models, not only empirical identification of relationships through data-driven analyses [6,7].

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An earlier generation of statistical mechanical models falling under the rubric of complexity science focused on exploring generative processes for complex emergent behaviors using relatively simple, data-agnostic, non-case-specific representations. Seminal examples included the Bak–Tang–Wiesenfeld (sandpile) cellular automaton, loosely corresponding to landslide occurrence, and Schelling's highly conceptualized agent-based model of urban segregation [8,9]. This modeling philosophy was not intended to reproduce specific instances of specific behaviors and did not attempt to directly analyze or replicate any specific observational datasets. Nevertheless, these models were conceptually tied to real-world phenomena and capable of suggesting hypotheses to help explain them, particularly vis-a-viz so-called stylized facts, that is, broad generalizations of previously discovered and widely noted anecdotal or empirical findings such as (for example) the extraordinarily wide prevalence of 1/*f* noises in physical, biological, and social systems [8,10,11]. A central result from such lines of investigation was that particular types of surprisingly simple mathematical models could prove capable of generating highly complex emergent behaviors. This breakthrough was important to forming and popularizing the field of complex systems science generally and, in certain cases like the Schelling model, computational sociology specifically.

In the spirit of these earlier models, this article presents an original representation of bilateral conflict and cooperation employing an agent-based modeling framework with some straightforward and reasonable assumptions around how certain social interactions might be mechanistically conceptualized. The model is abstract and generic, we concentrate on numerical experiments to trace out some aspects of model dynamics without direct analysis of observational datasets, and no claim is made for its superiority over any other model in replicating or predicting any particular phenomenon. Rather, its main interesting feature may be, in some sense, its generality: with only slight parameter perturbations, it reproduces, or produces dynamics that closely approximate, two different types of emergent complex behaviors that are widely important across the physical, life, and social sciences, and it appears to reproduce certain stylized facts around at least two important known forms of societal-scale conflict and cooperation. The number of existing models capable of providing generative mechanisms for any of these various behaviors is small, and to our knowledge no existing model can do so for all these phenomena. Typically, such generality is viewed by the physics community as a positive attribute. The outcomes may thus also reinforce the potential value of the earlier generation of mechanistic approaches (mentioned above) as a complementary approach to data science-based methods for exploring knowledge gaps around identifying and understanding underlying generative mechanisms in complex systems.

#### 2. Method

In this streamlined representation of bilateral conflict and cooperation, two partially coupled lattices represent two distinct but interacting tribes, and each cell in each of these grids is an agent which takes on one of three personality types. Scale is arbitrary: the two populations may be villages or nations, for instance, and an agent may be a person or some larger but distinct autonomous or semi-autonomous social unit. Agent locations within each grid represent social juxtapositions and separations, which can, but need not, correspond to geographical distance. Similarly, the two populations may stand geographically apart, or instead represent two spatially commingled but culturally independent identities, such as two strongly distinct and potentially conflictive or cooperative linguistic, ethnic, faith, or political groups within a single geographic entity like a city or region. For agent (*i*, *j*) within population *p* at time step *t*, define a cooperativity index,  $\phi(i,j,p,t)$ . That agent's willingness to cooperate over some issue with the other population as a whole is indicated by  $\phi > 0$ , the converse is denoted by  $\phi < 0$ , and  $\phi \sim 0$  indicates net indifference, which may variously reflect ambivalence, apathy, or compromise. The index parameterizes perceptions regarding the value and desirability of cooperation or conflict, implicitly encapsulating economic utility, loyalty to one's own population, empathy for the other population, anticipation that the other population will return either cooperative or hostile intent and action, elaborate political machinations, positive or negative personal histories, and so forth.

Individualists (personality type 1) are driven by independent free will. Each is represented as a stationary Markov process:

$$\phi(i, j, p, t) = \alpha \phi(i, j, p, t - 1) + \varepsilon(i, j, p, t)$$

that is, a first-order autoregressive model with serial correlation coefficient,  $0 < \alpha < 1$ , and white-noise forcing,  $\varepsilon$ , drawn from a fixed probability distribution independently for each *i*, *j*, *p*, *t*. This corresponds to a stochastically driven first-order linear differential equation, such as the linear reservoir equation in civil engineering or a type of Langevin equation in statistical mechanics; it represents a classical first-order linear time-invariant (LTI) systems modeling approach, and we may interpret it here as a reservoir of thought that can fluctuate toward different views on cooperation and conflict. That is, each individualist has a strong memory of its own beliefs but is also capable of large and unpredictable change over time and follows a path independent of other agents.

Networkers (personality type 2) are driven by social interactions with nearby individual agents within its own population, reflecting peer pressure, cultural diffusion, family traditions, and so forth. Such local social influence is routinely included in agent-based models; here, the agent polls opinions in its Moore neighborhood to determine degree of local consensus, and uses the proportion of socially adjacent agents sharing a positive or negative view regarding cooperation as the basis for its own opinion:

$$\phi(i, j, p, t) = \frac{1}{N} \sum_{k=i-1, l=j-1}^{k=i+1, l=j+1} sign[\phi(k, l, p, t)] \quad \forall k \neq i, \ l \neq j$$
(2)

where 3 < N < 8 is the number of neighbors to cell (*i*, *j*, *p*), depending on its location in the grid.

Reciprocators (type 3 agents) are driven by large-scale reactionary responses. Each reciprocating agent has a thresholded mirror response to the mean opinion of the other society:

$$\phi(i, j, p, t) = \begin{bmatrix} 1, & \mu(p', t) > \phi_{crit} \\ \phi(i, j, p, t - 1), & -\phi_{crit} < \mu(p', t) < \phi_{crit} \\ -1, & \mu(p', t) < -\phi_{crit} \end{bmatrix} \forall p' \neq p$$
(3)

where  $\mu(p',t)$  denotes the sample mean of  $\phi$  across all (i, j) for a fixed population p' at time t, and  $\phi_{crit}$  is a critical value. (3) is a tractable parameterization of the sociological and psychological fact that reciprocity is a fundamental aspect of human relationships. If a reciprocator believes the opposite tribe desires conflict, the agent will return the sentiment, whereas perception of friendly intentions from the other tribe inspires cooperation. Only strong feelings in one population elicit a response in the other population's reciprocators. A consequence is that general public opinion of one tribe about another can exhibit substantial persistence with little change in attitude over time, yet can also be punctuated by abrupt flip-flops; this representation is inspired by and consistent with the surprising rapidity in real societies with which long-time allies can become foes and vice versa. Some aspects of this formulation are reminiscent of the hysteresis suggested by [12]. Functionally, reciprocators serve as information pipelines that couple the two populations.

Many other kinds of personalities beyond the three invoked here are obviously possible, as are combinations of personality types, different ways of mathematically representing those personality types, and so forth. The model is not intended as a universal representation of cooperation and conflict processes. Nevertheless, the general notion of people whose opinions are personally guided by individualistic, networking, or reciprocating mentalities should be immediately recognizable from everyday experience and are basic concepts in social science and psychology, and it is interesting that a model based solely on relatively straightforward interactions between agents having these three personality types can produce a very wide range of societal-scale dynamics relevant to real-world systems, as described in the next section.

The model was implemented as follows. Agents are each assigned one of the three foregoing personality types on a random basis with probabilities  $P_{ind}$ ,  $P_{net}$ , and  $P_{rec}$ ;  $P_{ind} + P_{net} + P_{rec} = 1$ . Simulated synchronized execution was used [13]. Unless otherwise noted, each grid was 20 × 20;  $\alpha = 0.9$  and  $\varepsilon \sim N(0,1)$ , such that the cooperativity of individualists is a mean-reverting process with a long-term mean of 0, indicating in turn that the model is not intrinsically biased toward either conflict or cooperation;  $\phi_{crit} = 1/3$ ; initial conditions are random, specifically,  $\phi(i, j, p, t = 1) \sim U$  [-1,1]; and the simulation was run 5000 timesteps. Five scenarios were considered, consisting of Monte Carlo simulation experiments having slightly different parameter values or initial conditions: (A)  $P_{ind} = P_{net} = P_{rec} = 1/3$ ; (B) as in (A) but with a homogeneous and extreme initial condition of  $\phi(i,j,p,t = 1) = 1$ ; (C)  $P_{ind} = 1$  for population 1,  $P_{rec} = 1$  for population 2; (D)  $P_{ind} = P_{net} = \frac{1}{2}$ ; (E) a set of simulations identical to (A) but with grid size increased to 30 × 30, 40 × 40, and 80 × 80. The model was constructed in a Matlab script written for the purpose [14].

#### 3. Results and discussion

We begin with a short summary of basic system behaviors to establish context. We then consider the full set of simulations specified above, focusing our discussion on four specific behaviors of particular interest.

#### 3.1. General dynamics

Each personality type makes a unique contribution to overall system dynamics. Consider outcomes from simulation (A), having the agent type pattern shown in Fig. 1. Red-noise forcing provided by individualists drives the system's timevariability, but individualists also yield stability insofar as their mean value provides a goal toward which system-wide mean cooperativity across both populations,  $\mu$ , attempts to converge. Individualists socially assert their influence by imparting occasional large random fluctuations capable of flipping reciprocators to a new state, which in turn pulls the entire populations into a new state (e.g., phase transition-like event at  $t \sim 3450$  in Fig. 2) or by locally spreading their temporal fluctuations through adjacent networkers, where present (Fig. 3(a)). Such individualists correspond to opinion leaders. Conversely, if a particular individualist is not fortunate enough to contribute a large random fluctuation during the simulation, which can be seen as its lifetime, or to have networkers as neighbors, it may have little systematic influence (Fig. 3(b)). This corresponds to individualists that do not experience strong personal positive or negative sentiment around bilateral cooperation, or that are socially isolated so that their innovations, though potentially large, are not shared. On their own, networkers contribute little to the overall direction taken by the system, but they are central to formation of relationships and therefore the system's internal structure, spreading opinions beyond their source (Fig. 3(c)). They can be seen as opinion discoverers and communicators. Reciprocators are reactionary elements that can contribute sudden system regime changes, if a brief stochastic fluctuation driven by individualists in aggregate and spread by networkers is sufficiently large to drive the reciprocators into an alternative state (at  $t \sim 3450$  in Fig. 2), but also stability, holding constant if this random forcing is insufficiently large and widespread to kick reciprocators into an alternative state (as in simulation (B), shown in Fig. 4 and discussed further below). Their correlation structure exhibits distant but powerful linear associations with all the other reciprocators in both populations (Fig. 3(d)). Indeed, once an initial condition is



Fig. 1. Distributions of agent types within the two adjacent populations for simulation (A). Type 1: individualists; type 2: networkers; type 3: reciprocators.

overcome, variations in the cooperativity index across all reciprocator agents in both populations appear to march in lockstep, as they all mutually agree to a condition of inter-population conflict or cooperation at any given time (Fig. 2).

Time required for the system to evolve from a fully ordered or fully disordered initial condition exhibits a threshold response, depending on the initial state. Simulation (A) shows (Fig. 2) that the random, zero-mean, strongly heterogeneous starting state is forgotten within a few simulation timesteps.  $\mu$  is dynamic, moving rapidly to a quasi-equilibrium of random fluctuations (driven by collective effects of the random actions of many individualists as spread primarily, in this case, by networkers) about a stable mean (determined by the mean of  $\varepsilon$ ), or flipping to a new state when a random individualist-driven perturbation triggers the reactionaries to induce a phase transition-like event at  $t\sim$ 3450 (as discussed above). Conversely, simulation (B) shows (Fig. 4) that for an identically parameterized model except applying a homogeneous and extreme initial condition, random perturbations from individualists are insufficiently large to fully expel the system from its starting state over the course of the numerical experiment. While  $\mu$  immediately drops from its initial value of 1 due to the effects of individualists driven by mean( $\varepsilon$ ) = 0, and the networkers that spread this individualist information, it remains elevated at roughly 0.6 because the fluctuations are not large enough to change the mind of reciprocators, which remain frozen at  $\phi = 1$ .

#### 3.2. Emergent phenomena

If each population contains an even mix of individualists and networkers without reciprocators (simulation D), complex network topology evolves. Defining a graph theoretic link to exist between two agents anywhere system-wide if the absolute value of the linear Pearson product-moment correlation coefficient between their individual  $\phi(t)$  indices exceeds 0.5 [20], the resulting degree distribution is consistent with power-law scaling (Fig. 5a; see caption for analysis details). Breaking down outcomes by agent type (Fig. 5b) we see it arises from the superposition of the degree distributions of type 1 agents, who are in general poorly connected, and type 2 agents, who show a broader range of connectivity. A handful of networker-type agents effectively spread the influence of neighboring individualists, or of neighboring networkers in turn under the influence of adjacent individualists and so forth, forming super-nodes. These correspond to opinion leaders,



**Fig. 2.** Time series generated by simulation (A). Top two panels show mean  $\phi$  across all agents,  $\mu$ , across all agents only in population 1,  $\mu(p_1)$ , across all agents in population 2,  $\mu(p_2)$ , and across all individualists, networkers, reciprocators or reactionaries in both populations ( $\mu(ind)$ ,  $\mu(net)$ , and  $\mu(rct)$ , respectively). Middle panel shows  $\phi(ij,p,t)$  dynamics of six individual agents, with  $\mu$  repeated for reference; sites I and II are randomly selected individualists in populations 1 and 2, respectively; sites III and IV are randomly selected networkers in populations 1 and 2; sites V and VI are reciprocators in populations 1 and 2. Bottom two panels are similar to the top two panels but illustrate the standard deviations,  $\sigma$ , at a given time, t, rather than the mean.

discoverers, and communicators. Most agents, however, have far more limited social influence. By common definitions these characteristics correspond to a scale-free network, with a few highly connected agents and many poorly connected agents, and are thought to match real societies in certain respects [21].

If the two populations are instead asymmetrically parameterized, with one consisting entirely of individualists and the other of reciprocators (simulation C), radically different behaviors emerge: large aperiodic fluctuations, with individualists in one population sporadically provoking (when their collective stochastic fluctuations are sufficiently large) the reactionaries in the other population to repeatedly flip their state. This self-organized criticality is reflected in the emergence of power-law scaling (Fig. 5c and caption) in the spectral density function of  $\phi$  averaged across all agents in the system at a given time,  $\mu(t)$ , which appears to be traceable to superposition of aperiodic but generally high-frequency variability among type 1 agents and aperiodic but generally lower-frequency variability among type 3 agents. This scalefree (commonly referred to as 1/f, or more correctly  $1/f^{\beta}$ ) Fourier power spectrum over at least three to four orders of magnitude (Fig. 5c) corresponds to fractal dynamics, closely related to the Hurst effect and long-term memory. Such phenomena are commonly encountered in complex systems across the physical, life, and social sciences, with general implications including rarity of large events and preponderance of small events, absence of a characteristic timescale, and clustering of extreme events. This outcome seems to broadly align with a large number of stylized facts about conflict and cooperation and, in particular, meshes naturally with the now-widespread risk assessment concept of black swan events [22].



**Fig. 3.** One-point pairwise correlation maps for simulation (A). (a) Linear Pearson product-moment correlation coefficients between  $\phi(t)$  of a randomly selected individualist in population 2 (site II in Fig. 2) versus  $\phi(t)$  of every other agent. (b) As in (a) but for a randomly chosen individualist in population 1 (site I in Fig. 2). (c) For a networker in population 1 (site III in Fig. 2). (d) For a reciprocator in population 1 (site V in Fig. 2; results are identical for site VI).

We emphasize that fractal dynamics and scale-free networks are emergent, not designed, behaviors in this model. It is easy to construct a non-physical stochastic model generating a  $1/f^{\beta}$  power spectrum. However, relatively few models explicitly incorporating some level of mechanistic process, like a handful of statistical mechanical models including the Bak–Tang–Wiesenfeld (sandpile) cellular automaton and slider-block earthquake models, or certain detailed models of geophysics, solid state physics, and econometrics involving superpositions of simpler linear memory processes, can



Fig. 4. As in Fig. 2, but for simulation (B).

generate it as an emergent phenomenon [8,23–26]. Similarly, scale-free networks are often inferred empirically from direct analysis of observational data, and agent-based models are commonly built on networks statistically designed to have certain properties, including scale-free degree distributions. Again, however, there are only a few process models, such as the preferential attachment model, that generate a scale-free network as an emergent behavior [21]. In addition to adding to the short library of mechanistic generating models for these widely important complex behaviors, to our knowledge this model appears the only capable of producing both; which occurs depends only on the values set for a few simple parameters (the relative proportions of the three personality types), without changing the structure or any other parameters of the model.

#### 3.3. Possible clues to two sociological puzzles

A system containing even mixtures of all three agent types in both populations (simulation A) may suggest an explanation for Richardson's 1948 discovery that war size, measured by casualty numbers, follows a monotonically decreasing, long-tailed distribution, thought to approximately obey a power law [27–29]. A histogram of conflict sizes from the model,  $abs(\mu) \forall t \mid \mu < 0$ , exhibits this general form (Fig. 5d). Our mechanistic model therefore reproduces the broad features of Richardson's empirical discovery. While it has the disadvantage of offering less specific interpretations than the military logistics and geopolitical model put forth by [28] to similarly explain war size distribution, it may have the benefit of potentially speaking to additional, related phenomena identified by Richardson, like similar distributions for murder rates [27]. Two corollaries are that the conflict size distribution is not precisely described by a power law, and that the full distribution of  $\mu$ , including both cooperation and conflict events, exhibits a more complex, bimodal shape; this raises an intriguing possibility that certain empirically inferred stylized facts, like Richardson's observations, might



**Fig. 5.** Some emergent behaviors of the model. (a) Heavy-tailed degree distribution of network spontaneously formed by simulation (D), showing observed frequencies, best-fit least-squares log-space regression line (t-test p << 0.01), and 95% confidence limits estimated as  $\pm 1.96$  standard error. (b) Degree distributions for simulation (D), showing results for all agents, only individualists, and only networkers. Recall  $P_{rec} \equiv 0$  for this simulation. (c) Power-law Fourier spectrum of  $\mu$  over about four orders of magnitude for simulation (C), showing observed spectrum, best-fit least-squares log-space regression line (t-test p << 0.01), and 95% confidence limits estimated as  $\pm 1.96$  standard error. (d) Histogram of conflict event magnitudes,  $abs(\mu) \forall t \mid \mu < 0$ , in simulation (A). Fundamental definitions and rigorous statistical detection of complex system phenomena, including scale-free networks and 1/*f* spectral scaling, in observational datasets representing complicated, multi-faceted, and incompletely understood real-world systems can be intricate and subject to debate [e.g., [15–19]]. Our analysis is sufficient for identifying the behaviors.

in part reflect a focus on cataloging only one kind of event (i.e., conflicts), and a more encompassing view of conflict and cooperation as a continuum (i.e.,  $\phi$ ) could prove instructive and lead to different outcomes.

A suite of simulations each using an even mix of personality types but with progressively larger population sizes (simulation E) may provide insights into another puzzle. The largest instantaneous system-wide conflict or cooperation event, max[ $abs(\mu)$ ], exhibits an approximate power-law relationship ( $R^2 = 0.91$ ) with grid size. This nonlinear finite-size effect appears to reflect the net outcome of two considerations: the law of large numbers, such that at a given time step, the larger the lattice dimensions and therefore the number of samples (agents), the closer the sample mean ( $\mu$ ) across those lattices approximates the underlying mean of zero in the AR(1) process describing individualists, rather than demonstrating some large random fluctuation; and accordingly, for larger societies, the system is less likely to push reciprocators into extreme conflict or cooperation states, allowing the mean state to converge around that individualist

mean of zero. This may correspond to some basic known features of real social groups as a function of their size, such as the inertia of policy and procedure in a large organization, even when its members have a wide diversity of individual opinions, versus the dynamism and instability of small groups.

This decrease in the magnitude of the largest conflict or cooperation event with increasing social scale may prove informative to a seemingly paradoxical empirical result in economic, energy, and environmental security. Former World Bank vice-president Ismail Serageldin remarked that the wars of the 21st century would be fought over water, due primarily to increasing water demand under population growth and secondarily to increasing water supply uncertainty under climate change; concern about potential for such so-called water wars is now widespread. However, social scientists have observed a more nuanced reality: local-to-regional scale water challenges exhibit wide-ranging outcomes, spanning enthusiastic cooperation to occasional violent conflict, yet water management at international scales tends to stabilize toward grudging cooperation and non-violence [30–32] (for background see [33,34]). This consistent reduction in water conflict severity with increasing social and geographical scale, deduced largely from qualitative or semi-quantitative observations, is consistent with our theoretical approximate power-law scaling relationship, which is in turn a quantitatively testable model prediction that could guide objective high-quality experimental data collection in this comparatively data-sparse field.

#### 4. Conclusions

Spontaneously emergent complex behaviors in a novel agent-based model of bilateral conflict and cooperation include, according to common definitions of these phenomena, fractal dynamics in overall system cooperativity, and a scale-free network of pairwise interactions between agents. Complex emergent behaviors like these are ubiquitous in natural and artificial systems, including sociological systems, and have profound consequences, such as prevalence and clustering of extreme events and network stability. Our model adds to the comparatively brief list of mechanistic generative mechanisms for these phenomena and to our knowledge is the only model that produces both. It also generates other nonlinear behaviors, such as threshold responses to initial conditions. The model additionally suggests hypotheses that may help explain two puzzles of social science: Richardson's 1948 empirical discovery of monotonically decreasing long-tailed distributions of war size; and observed differences, between local versus international scales, in the prevalence and severity of conflict and cooperation over increasingly scarce water resources. These explanations constitute quantitatively testable hypotheses about the origins of such stylized sociological facts, and as such they may in turn suggest specific directions for observational data collection, and perhaps provide guidance to developing more complete and detailed mechanistic models of these behaviors.

An interesting feature of the model is that adjusting the values of just a few of its parameters produces a diverse array of modes of social dynamics. That net collective dynamics can be radically altered without any change whatsoever in underlying process and structure suggests a generally applicable and data-agnostic model capable of simulating, at least at some conceptual level, a diverse set of social conditions. To whatever extent the model reproduces actual bilateral conflict and cooperation, this result may also imply that fundamental changes in relationships between pairs of societies – e.g., transitioning from war-like to peaceful standings, or vice versa – might conceivably be attained by incremental, non-structural, and comparatively easily manipulated changes in their properties.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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