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MEAN AREAL PRECIPITATION FOR DAILY HYDROLOGIC MODELING IN MOUNTAINOUS REGIONS[1]

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ABSTRACT: A procedure using detrended kriging has been developed to calculate daily values of mean areal precipitation (MAP) for input to hydrologic models. The important features of this procedure that overcome weaknesses in existing MAP procedures are: (1) specific precipitation-elevation relationships are determined for each time period as opposed to using relationships based on climatological averages, (2) spatial variability is incorporated by estimating precipitation for each grid cell over a watershed, (3) the spatial correlation structure of precipitation is explicitly modeled, and (4) station weights for precipitation estimates are determined objectively and optimally. Detailed cross-validation testing of the procedure was done for the Reynolds Creek research watershed in southwestern Idaho. The procedure is suitable for use in operational streamflow forecasting.

(KEY TERMS: precipitation; surface water hydrology; modeling/statistics; kriging.)

INTRODUCTION

Conceptual hydrologic models are used for a variety of tasks in watershed simulation, water resources management, and streamflow forecasting. Examples of these models include the Stanford Watershed Model (Crawford and Linsley, 1966), the snow and soil moisture accounting models within the National Weather Service River Forecast System (NWSRFS) (Anderson, 1973; Burnash *et al.*, 1973), the Streamflow Synthesis and Reservoir Regulation (SSARR) model (U. S. Army Corps of Engineers, 1987), the Precipitation-Runoff Modeling System (PRMS) developed by the U. S. Geological Survey (Leavesley *et al.*, 1983), and the HBV model developed by the Swedish Meteorological and Hydrological Institute (Bergström, 1992). One of the primary inputs to conceptual hydrologic models is precipitation; a time scale of one day is commonly used. Whether the watershed is modeled as a single unit or divided into sub-areas, mean areal precipitation (MAP) over one or more parts of the watershed is required.

The classical techniques of estimating precipitation at a point include the normal-ratio method and inverse-distance-squared weighting method; the classical techniques for estimating MAP from point measurements include Thiessen polygons and the isohyetal method (Linsley *et al.*, 1975). While these techniques are relatively simple and straightforward, they have simplistic assumptions about the spatial correlation and variability of precipitation, do not handle orographic effects well, can be subjective, and are not necessarily optimal. The National Weather Service uses a procedure that improves upon these classical techniques by using isohyetal maps of mean annual or seasonal precipitation to evaluate the climatological average orographic effect (Anderson, 1988), but it, too, has limitations in that it requires subjective selection of relative station weights, and it assumes that the orographic effect is the same as the climatological average for all storms.

A more recent technique for estimating MAP is the use of kriging, an optimal spatial interpolation procedure for estimating the values of a variable at unmeasured points from nearby measurements. It was first developed for use in the mining industry and has subsequently found widespread use in geology and hydrology (where it is often called "geostatistics"). Many papers exist describing the theory and applications of kriging; ones relating to precipitation include Delfiner and Delhomme (1975), Chua and Bras (1982), Creutin and Obled (1982), Bastin *et al.* (1984),

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Bastin and Gevers (1985), Lebel *et al.* (1987), Lebel and Laborde (1988), Dingman et al. (1988), Hevesi *et al.* (1992a,b), and Phillips *et al.* (1992). Textbook descriptions of kriging appear in Bras and Rodriguez-Iturbe (1985), McCuen and Snyder (1986), and Journel (1989). The aspects of kriging theory relevant to this work are summarized in the Appendix. Kriging is objective, statistically rigorous, and performs as well as or better than other estimation techniques for precipitation (Tabios and Salas, 1985). Day *et al.* (1989) and Day (1990) used a similar technique to calculate mean areal snow water equivalent.

All of the previous applications of kriging to precipitation data dealt with estimating MAP for a single storm or for annual totals, and none attempted to develop a daily time series of MAP. With the additional considerations required for daily MAP, the procedure is described below.

KRIGING PROCEDURE

Kriging can be used to estimate precipitation for the cells on a rectangular grid throughout a watershed, and these values can be arithmetically averaged to obtain MAP. The grid is most conveniently established using a geographic information system, although it can also be done manually using maps. Each grid cell is characterized by its location (latitude and longitude or rectangular coordinates) and elevation. The number and size of grid cells should give an adequate representation of the watershed's topography and should closely approximate the area-elevation relationship derived from complete topographic data. For example, one basin used in this work had 200 grid cells (2 minutes latitude by 2 minutes longitude) for a 1680 km^2 area.

Grid cell precipitation estimates are obtained from a weighted sum of measurements at a number of stations in or near the watershed, where the sum of the weights is unity. The weights to be used on each measurement to estimate the precipitation at a grid cell are determined by solving a system of linear equations, the coefficients of which are a function of the distances among the locations of the gages and the location of the grid cell (see Appendix). Each grid cell, then, has its own unique set of weights (summing to unity) to be applied to the precipitation measurements. These estimates are optimal in that the spatial correlation structure is explicitly modeled (via the semivariogram), and the weights on the measurements are derived so as to give minimum error variance in the estimate.

Orographic Effects

In mountainous areas, orographic effects complicate the estimation of MAP. Kriging requires a stationary field for estimation; that is, there must be no systematic spatial trend or "drift" in the mean or variance of the process. This is not the case in mountainous areas, where precipitation generally increases with elevation and is influenced by storm direction and topographic aspect. Chua and Bras (1982) used two methods for dealing with this nonstationarity, one involving generalized covariances, and the other subtracting a precipitation-elevation trend from the data before performing the kriging (detrended kriging). Dingman *et al.* (1988) used detrended kriging, Hevesi *et al.* (1992a,b) used cokriging (with elevation as the covariate), and Phillips *et al.* (1992) used both detrended kriging and cokriging. Detrended kriging was used in the procedure reported herein because it gave better results for Chua and Bras (1982) and Phillips *et al.* (1992), and it is simpler and more straightforward than the other two methods for dealing with nonstationarity. Following Chua and Bras (1982), Dingman *et al.* (1988), Phillips *et al.* (1992), and Daly *et al.* (1994), linear precipitation-elevation relationships were used for the detrending.

Besides elevation, however, other factors, such as topographic aspect, can be important in affecting precipitation. For example, Hanson (1982) found that separate linear relationships were required for sites on the windward and leeward sides of topographic barriers in the Reynolds Creek watershed in southwest Idaho. Again in Idaho, Winters *et al.* (1989) used a lifting index based on topography and prevailing wind direction to help describe spatial variability in mean annual precipitation. Daly *et al.* (1994) used spatially smoothed elevations ("orographic elevation") instead of actual (point) elevations, and they grouped stations according to similarity in aspect. Dividing the watershed into regions of similar orographic regime was used in the procedure developed here; refinements in detrending the precipitation field is an area worthy of continued investigation.

Most existing hydrologic models make use of average orographic relationships for either the entire year or for two or more periods of the year (e.g., Anderson, 1988). That is, it is assumed that the variation of precipitation with elevation always adheres to these historical relationships. This is not necessarily true, however, because the storms in a given period for one year could have different directions and intensities from the storms in the same period in another year, yielding different precipitation-elevation relationships. It is important, then, that the time-varying nature of orographic effects be included in the MAP procedure. This, of course, requires a data network with a good range in elevation. This exists in the western United States, where high elevation precipitation data from the USDA Soil Conservation Service's SNOTEL network, together with National Weather Service cooperative network sites in the lower elevations, make it possible to determine precipitation-elevation relationships fairly accurately.

The MAP procedure developed here calculates a precipitation-elevation relationship for each time period being modeled. It was hypothesized that daily precipitation-elevation relationships might be subject to large fluctuations and instability, so to ensure that the relationships are robustly estimated, the precipitation data are aggregated into consecutive periods of 7, 14, or 28 days in length, or aggregated into storm periods (a storm being defined as consecutive days where at least one station had precipitation). Daily precipitation residuals are calculated by subtracting the precipitation-elevation trend from the precipitation for each day within the aggregation period. These residuals are the values used in kriging. The choice of an aggregation period depends on the precipitation regime (frequency and amount of precipitation, consistency of storm tracks, etc.). At this time, the effect of the choice of aggregation period is not entirely clear; some comparisons will be given later. Future work should clarify this aspect of the procedure.

Spatial Correlation: the Semivariogram

The spatial correlation structure of precipitation is modeled in kriging by the semivariogram. McCuen and Snyder (1986) list several functional forms commonly used to model the semivariogram, including linear, exponential, and spherical; the pertinent aspects of semivariograms are explained in the Appendix. After examining many empirical semivariograms calculated from detrended daily precipitation data in several watersheds in the West (2700 km^2 or less), a linear semivariogram appeared to be adequate to model the residuals. These empirical semivariograms exhibited significant scatter around a general upward trend, and, at this spatial scale, a flattening of the relationship could not be detected. Semivariograms that flatten out as the distance between stations increases are more commonly used [e.g., Chua and Bras (1982) used a spherical semivariogram], although Karlinger and Skrivan (1981) used a linear semivariogram to describe mean annual precipitation in Montana and Wyoming.

A convenient property of a linear semivariogram is that the kriging weights are independent of the slope and intercept of the line. Because the coefficients in the kriging system of equations (see Appendix) for two different linear semivariograms are linear functions of one another, the solution does not change (except for the Lagrange parameter, which is related to the uncertainty of the estimates). Since the semivariogram is a function of distances between stations, the kriging weights can be obtained by using these distances themselves as the coefficients in the kriging system of equations. If there are no missing data at the precipitation stations, then the kriging weights can be calculated once for each grid point to be estimated, and this set of weights can be used for all periods and years.

Summary of Calculation Procedure

To summarize the above discussion, the steps of the calculation procedure are as follows:

(1) Prepare daily time series of precipitation data for stations in or near the watershed of interest. Include both low and high elevation stations. Missing data can either be estimated, if an accurate estimating technique is available, or they can be left as missing, in which case the precipitation-elevation relationships and kriging weights can be calculated using only the stations that are available.

(2) Establish a grid over the watershed, preferably using a geographic information system, or alternatively, using maps. Obtain the latitude and longitude (or other location coordinates) and elevation for each grid cell.

(3) Calculate the kriging weights to be used for each grid cell for the case when all precipitation stations have data. This assumes the use of a linear semivariogram.

(4) Choose an aggregation period. Typical choices are 7, 14, or 28 days, or storm periods.

(5) For each period, calculate average daily precipitation at each station, using only days for which at least one station had precipitation ("wet" days). Calculate the linear regression of average daily precipitation (dependent variable) versus station elevation (independent variable).

(6) For each wet day within the period, subtract the linear precipitation-elevation trend from the precipitation observations to obtain the residuals. For each day where all stations had zero precipitation ("dry" days), set MAP equal to zero, and do not process these days further.

(7) For each wet day within the period and for each grid cell, calculate the estimated grid cell residual by multiplying the precipitation station residuals by the kriging weights and summing. If one or more precipitation stations have missing data, the kriging

weights must be recalculated to use only the stations that have data; otherwise, the weights calculated in step 3 can be used.

(8) For each wet day within the period and for each grid cell, add the linear precipitation-elevation trend to the grid cell residuals, based on the elevations of the grid cells, to obtain the estimated grid cell precipitation.

(9) For each wet day within the period, arithmetically average the grid cell precipitation values to obtain MAP.

If the watershed is subdivided, repeat steps 5-9 for each sub-area, using only the grid cells that fall within each sub-area. This results in a MAP time series for each sub-area.

APPLICATION TO REYNOLDS CREEK

The MAP procedure described above has been applied to several watersheds in the western United States. Although results appear reasonable, it is impossible to tell whether the MAP values are correct because the true MAP is unknown. One can, however, obtain some idea of the accuracy of the procedure by doing a cross-validation analysis; that is, a precipitation station can be removed from the data set, and the procedure can be used to estimate precipitation at the removed station from the remaining stations. By returning this station to the data set and removing another, the process can be repeated to estimate all precipitation stations, and error statistics can be calculated.

An excellent data set with which to make these tests is from the Reynolds Creek research watershed in southwest Idaho, which is operated by the USDA Agricultural Research Service (Robins *et al.*, 1965). The Reynolds Creek watershed covers 233 km^2, with an elevation range of 1100 to 2200 m (Figure 1). Mean annual precipitation ranges from 250 mm in the northeast part of the watershed to 1150 mm at the highest elevations in the southwest (Stephenson, 1977). A daily data set for 12 stations (Figure 1) over the period 1962-1990 was available for this study.

Designation of Watershed Sub-Areas

The first step in the analysis was to determine if the watershed needed to be divided into sub-areas due to variations in orographic regimes. Mean daily precipitation for wet days was calculated for each station for each of 13 28-day periods (leftover days were lumped into the last period) over the 1962-1990 data set. These values were then plotted versus elevation; examples are given in Figure 2. Although there was some variability among the periods, two groups of stations were evident in most of these plots, particularly during the winter: Group 1, consisting of stations 057, 076, 095, 116, 144, 155, 163, and 174; and Group 2, consisting of stations 127, 147, 167, and 176. Group 1 is located in the western part of the watershed, and Group 2 is in the eastern part; this is similar to the regions described by Hanson (1982). In some of the cross-validation tests described below, comparisons are made between the estimation errors from using the groupings and the errors from lumping all stations together.

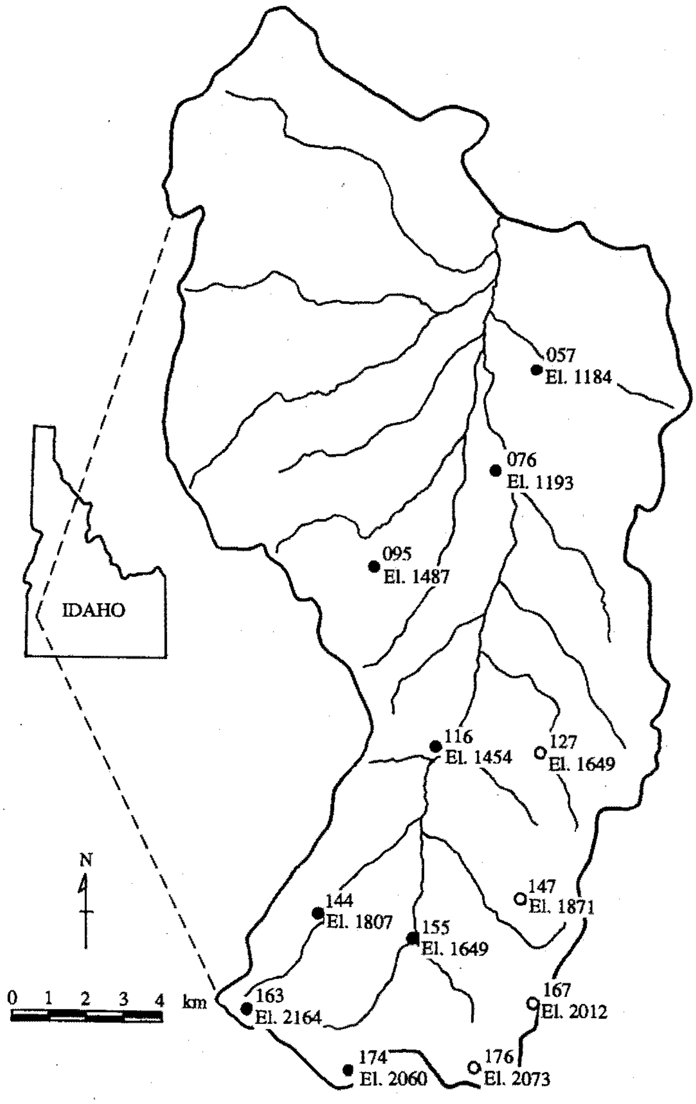

Figure 1. Reynolds Creek Watershed and Precipitation Gages Used in This Study (elevations in meters). Solid circles are Group 1 stations; open circles are Group 2 stations.

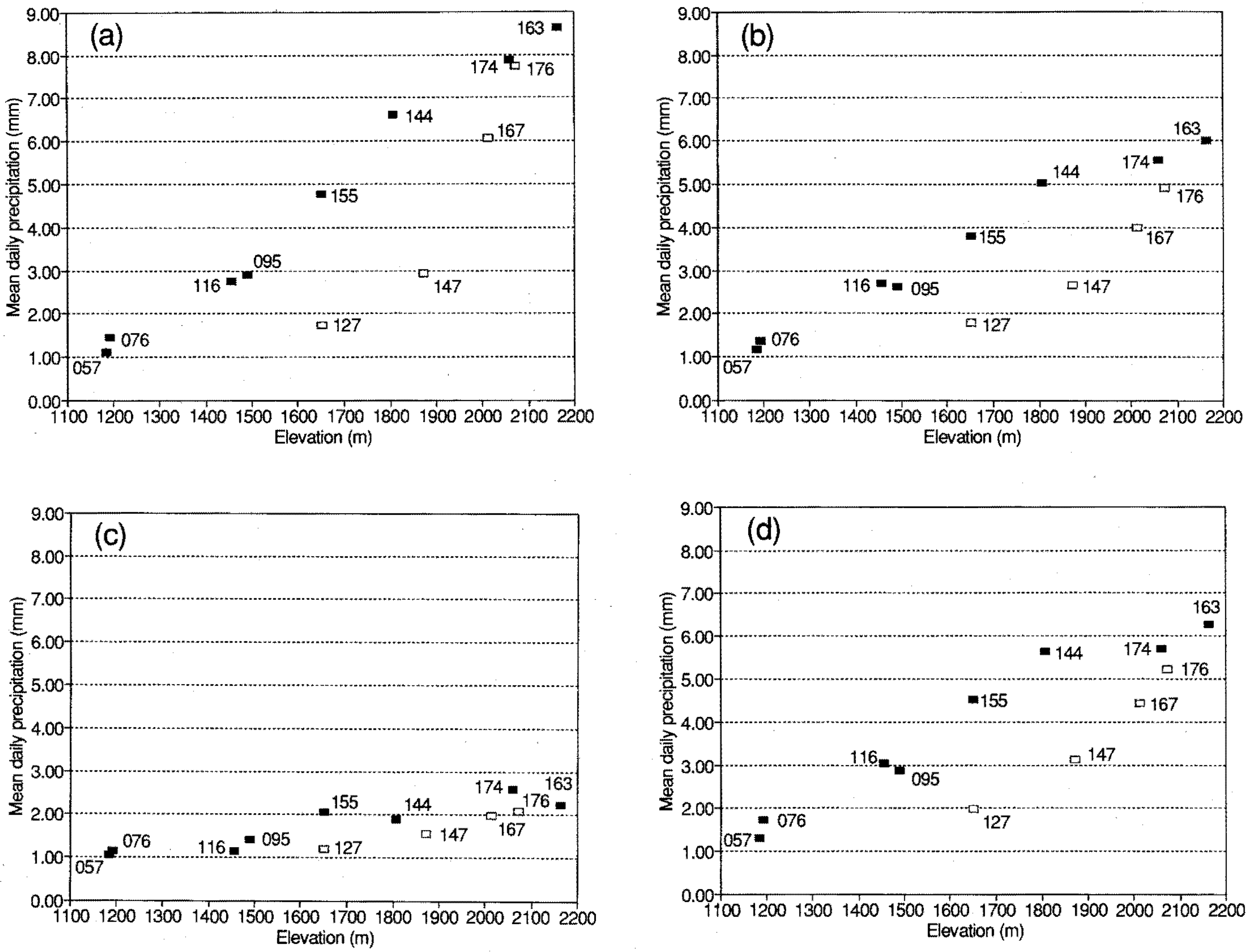

Figure 2. Precipitation-Elevation Relationships for Wet Days at Reynolds Creek, 1962-1990. (a) January 1-January 28 (period 1 of 28-day periods). (b) March 26-April 22 (period 4 of 28-day periods). (c) July 16-August 12 (period 8 of 28-day periods). (d) October 8-November 4 (period 11 of 28-day periods). Shaded rectangles are Group 1 stations; open rectangles are Group 2 stations.

Tests of MAP Procedure

Several cross-validation tests were conducted to investigate the following:

(1) Whether the kriging station weightings provided greater accuracy than simply using equal station weighting.
(2) The effect of different aggregation periods: 7, 14, and 28 days, and storm periods.
(3) The effect of grouping the stations versus lumping all stations together.
(4) How well the spatial average of the observed data for the 12 stations compares with that using the cross-validation estimates.

Three error statistics were used in these tests for accuracy comparisons: (1) mean absolute error of daily precipitation (mm), (2) the percentage of days when the observed precipitation was zero but the estimated precipitation was nonzero; and (3) the percentage of days when the observed precipitation was nonzero but the estimated precipitation was zero. Statistics (2) and (3) will be referred to as misspecified days. The results of each test are described below.

Kriging vs. Equal Weights. The first question evaluated was to determine whether the distance weighting by kriging (based on a linear semivariogram) gave better cross-validation accuracy than using equal weights on all stations. Thirteen 28-day aggregation periods were used for this test, and stations were grouped. For all 12 stations, kriging gave smaller mean absolute errors (Table 1), and for 9 of the 12 stations, kriging gave fewer misspecified days (Table 2). Thus, there seemed to be important correlation structure in the precipitation residuals, and distance weighting was appropriate.

TABLE 1. Cross-Validation Mean Absolute Errors (mm/day) for Kriging vs. Equal Weights and Grouped vs. Ungrouped Stations Using a 28-Day Aggregation Period.

Station	Kriging Grouped	Equal Weights Grouped	Kriging Ungrouped
	Group 1		
057	0.303	0.832	0.286
076	0.257	0.717	0.262
095	0.567	0.636	0.594
116	0.477	0.588	0.442
144	0.800	0.990	0.685
155	0.617	0.635	0.558
163	1.051	1.657	1.065
174	0.932	1.377	0.854
Average	0.625	0.929	0.593
	Group 2		
127	0.657	0.756	0.617
147	0.705	0.803	0.801
167	0.661	0.868	0.658
176	0.940	1.260	0.763
Average	0.741	0.922	0.710
Overall Average	0.664	0.927	0.632

For the remainder of the cross-validation tests, kriging was used. The kriging weights used for the cross-validation estimates for each station are given in Table 3a (Group 1) and Table 3b (Group 2). Some of the weights in Group 1 had small negative values; this was an artifact of the redundancy of information contained in the stations. In most cases, there were two or three stations that carried the bulk of the weighting, and the rest of the weights were very small (both positive and negative), being primarily statistical noise.

Aggregation Period. Cross-validation tests were made for each of 52 7-day periods, 26 14-day periods, and 13 28-day periods (leftover days lumped into the last period) and for storm periods. Storm periods were defined as consecutive days during which at least one station had precipitation; MAP values for days between storms were set to zero, as all stations had zero precipitation.

The error statistics showed little difference among the four aggregation periods, that is, the values were very similar to those given in Table 1. Of the differences noted, no single aggregation period was clearly superior for a majority of stations. In this watershed, then, it appeared that the choice of aggregation period was not significant. This may not be true for other watersheds.

Station Grouping. Cross-validation tests were made for the 28-day aggregation periods to evaluate the effect of grouping stations. Results were mixed (Tables 1 and 2). Eight of the 12 stations had smaller mean absolute errors when left ungrouped, although the difference in the errors was generally of a small magnitude. Eleven of the 12 stations, however, had fewer misspecified days when the stations were grouped. It appears, then, that in this case, precipitation estimates are not particularly sensitive to differentiating between the two orographic areas. There could be at least two reasons for this. One may be that the precipitation-elevation relationships were less robustly estimated for the groups because fewer stations were used than in the ungrouped case. Another reason may be that as long as a reasonably good general orographic effect can be estimated, the distance weighting within the kriging algorithm is adequate to obtain realistic estimates of the site-specific residuals from the overall trend.

In most watersheds, the precipitation network is not nearly as dense as it is for Reynolds Creek, sometimes making it difficult to define areas of differing orographic effects very clearly. If, in fact, the MAP procedure is not particularly sensitive to the establishment of highly accurate, region-specific precipitation-elevation relationships, this gives hope that good MAP estimates can be obtained in applications where the best one can do is estimate general, basin-wide orographic effects.

Kriging Estimates of Spatially-Averaged Precipitation. Since the Reynolds Creek precipitation network is fairly dense, one could consider an arithmetic average of the precipitation at the 12 stations to be a reasonable estimate of true MAP. This could then be compared to an arithmetic average of the cross-validation estimates at each station to obtain some idea of the accuracy of the MAP estimation procedure for spatially-averaged values.

The above test was performed for the 29-year time series of total annual precipitation. The mean absolute error was 12.4 mm for cross-validation estimates made with grouped stations and 19.6 mm for ungrouped stations. Comparing these with the 29-year, 12-station mean annual precipitation (655 mm), these errors are 1.9 percent and 3.0 percent, respectively.

TABLE 2. Cross-Validation Misspecified Days for Kriging vs. Equal Weights and Grouped vs. Ungrouped Stations Using 28-Day Aggregation Periods.

	Obs. = 0, Est. > 0 (percent of days)			Obs. > 0, Est. = 0 (percent of days)			Total Misspecified (percent of days)		
Station	Kriging Grouped	Equal Weights Grouped	Kriging Ungrouped	Kriging Grouped	Equal Weights Grouped	Kriging Ungrouped	Kriging Grouped	Equal Weights Grouped	Kriging Ungrouped
				Group 1					
057	4.3	3.4	3.4	2.7	6.7	3.9	7.0	10.1	7.4
076	1.5	2.4	1.5	5.9	7.7	6.6	7.4	10.1	8.1
095	15.3	3.9	18.0	0.6	4.2	0.5	15.9	8.1	18.5
116	3.0	2.4	1.5	3.2	5.0	6.9	6.2	7.4	8.3
144	5.6	9.3	6.2	2.4	1.4	2.0	8.0	10.8	8.1
155	1.8	9.7	1.5	5.2	0.8	6.8	7.0	10.5	8.4
163	8.3	8.4	11.1	1.7	1.7	1.2	9.9	10.1	12.3
174	8.4	8.9	4.2	1.4	1.4	2.0	9.8	10.3	6.2
Average	6.0	6.1	5.9	2.9	3.6	3.7	8.9	9.7	9.7
				Group 2					
127	2.1	2.2	18.1	14.0	12.8	0.7	16.0	15.0	18.8
147	9.0	4.6	15.9	1.4	3.0	0.8	10.4	7.7	16.8
167	6.5	6.9	10.4	1.8	1.8	1.4	8.3	8.7	11.7
176	4.7	4.9	9.2	3.4	3.4	1.1	8.1	8.3	10.3
Average	5.6	4.7	13.4	5.2	5.3	1.0	10.7	9.9	14.4
Overall Average	5.9	5.6	8.4	3.6	4.2	2.8	9.5	9.8	11.2

TABLE 3a. Kriging Weights for Group 1 Stations.

Station Estimated	Estimator Stations							
	057	076	095	116	144	155	163	174
057		0.979	0.032	–0.007	–0.017	0.016	–0.006	0.004
076	0.578		0.349	0.094	–0.029	0.018	–0.018	0.009
095	0.029	0.525		0.376	0.134	–0.092	0.091	–0.061
116	–0.006	0.136	0.363		0.210	0.381	–0.058	–0.026
144	–0.009	–0.026	0.079	0.129		0.437	0.396	–0.006
155	0.009	0.018	–0.062	0.264	0.492		–0.092	0.371
163	–0.005	–0.022	0.076	–0.050	0.556	–0.114		0.560
174	0.003	0.012	–0.054	–0.024	–0.009	0.485	0.586	

The errors, however, were almost all of a positive sign, indicating a small bias in the estimates (estimated precipitation slightly larger than observed). The cause of this is not entirely clear, but there did seem to be a greater tendency for estimated precipitation to be non-zero on days when the observed precipitation was zero than vice versa. This imbalance was more prevalent in Group 1 stations. Whether this bias is specific to this watershed or is inherent in the procedure needs to be investigated further. In any event, the bias is quite small. Note, too, that in this test, the results from grouped stations were better than ungrouped stations, but the difference was small.

TABLE 3b. Kriging Weights for Group 2 Stations.

Station Estimated	Estimator Stations			
	127	147	167	176
127		0.994	0.004	0.002
147	0.423		0.515	0.062
167	0.001	0.446		0.553
176	0.002	0.089	0.909	

Temporal Variability of Orographic Effects

One of the advantages of this MAP procedure is that the precipitation-elevation relationships used are determined for the specific time period under consideration rather than using average relationships based on climatological means. The seasonal and annual variability of the orographic effect can be substantial, as shown in Table 4, which gives statistics for the slopes of the precipitation-elevation lines (mm / 10^3 m) used for 28-day periods with Group 1 stations. The seasonal variability is reflected in the changes in the mean slope through the year; the annual variability is reflected in the range, standard deviation, and coefficient of variation of the slopes for each period. Because of this variability, it is important to determine the orographic effect for each time period under consideration rather than assume the orographic effects always follow climatological patterns.

CONCLUSION

A procedure using detrended kriging has been developed to calculate daily values of MAP for input to hydrologic models, overcoming several weaknesses in existing MAP procedures. The advantages of this procedure include (1) specific precipitation-elevation relationships determined for each time period as opposed to relationships based on climatological averages, (2) spatial variability incorporated by estimating precipitation for each cell of a grid over the watershed, (3) spatial correlation structure explicitly modeled via the semivariogram, and (4) objective determination of station weights for precipitation estimates.

Future investigations will examine, for watersheds other than Reynolds Creek: (1) whether stations really need to be grouped if different orographic regimes exist in the watershed of interest; (2) the effect of different aggregation periods; and (3) determining if the procedure has an inherent positive bias. Another area to be investigated is the use of smoothed elevations and other topographic variables in the detrending. Smoothed elevations may in fact be better indicators of orographic effects, as suggested by Daly *et al.* (1994); other topographic variables, such as slope or aspect, may also be useful in explaining orographic effects. These issues should be resolved as the procedure is applied to other watersheds and compared to conventional MAP procedures.

With the increasing availability of geographic information systems, procedures that use gridded topographic data are far more feasible than in the past. Considering this, it is likely that procedures such as the one described herein will be used in routine operations, such as streamflow forecasting.

APPENDIX
KRIGING CALCULATIONS

The first step in kriging is to estimate the variogram. This is the function that describes the spatial correlation structure of the data. The variogram is the

TABLE 4. Statistics for Slope of Precipitation-Elevation Lines, 28-Day Periods, Group 1 Stations.

Period	Dates (non-leap year)	Mean (mm/10^3 m)	Range (mm/10^3 m)	Standard Deviation (mm/10^3 m)	Coefficient of Variation
1	January 1-January 28	8.350	1.300 - 19.000	4.075	0.49
2	January 29-February 25	7.558	2.825 - 16.050	3.258	0.43
3	February 26-March 25	5.842	0.900 - 12.050	2.475	0.42
4	March 26-April 22	5.342	2.033 - 9.600	2.133	0.40
5	April 23-May 20	3.283	–0.367 - 5.942	1.700	0.52
6	May 21-June 17	3.150	0.217 - 11.550	2.375	0.75
7	June 18-July 15	1.558	–1.767 - 4.867	1.733	1.12
8	July 16-August 12	1.708	–1.158 - 9.325	2.000	1.17
9	August 13-September 9	1.717	–2.050 - 7.658	2.275	1.33
10	September 10-October 7	3.033	0.000 - 8.692	2.375	0.78
11	October 8-November 4	5.342	0.958 - 13.733	3.225	0.60
12	November 5-December 2	8.217	3.050 - 15.883	3.117	0.38
13	December 3-December 31	6.533	1.542 - 12.125	2.858	0.44

variance of the differences between data values separated by a distance h and is calculated as follows:

$$2\hat{\tau}(h) = \frac{1}{n}\sum_{i=1}^{n}\left[Y_i(\mathbf{x}) - Y_i(\mathbf{x}+h)\right]^2 \qquad \text{(A1)}$$

where $2\hat{\tau}(h)$ is the sample estimate of the variogram, h is the distance between data sites, $\mathbf{x}$ is a vector in a two-coordinate system describing the spatial location of a data site, $Y(\mathbf{x})$ is the data value at point $\mathbf{x}$, and n is the number of site pairs separated by the distance h. When dealing with precipitation measurements, n is usually one; n is greater than one only if measurements are available on a regular grid, which may exist in other contexts. One can, however, group data pairs into distance categories to help smooth the estimated variogram; n would then be the number of pairs in each distance category.

The function $\tau(h)$ is called the semivariogram. The values of this function are what are actually used in the kriging calculations. The typical shape and features of a semivariogram are shown in Figure A1. The semivariogram is usually modeled by one of several analytic functions:

Linear: $\tau(h) = \tau_n + bh$ (A2a)

Power: $\tau(h) = \tau_n + bh^c$ (A2b)

Logarithmic: $\tau(h) = \tau_n + 3b\ln(h)$ (A2c)

Exponential: $\tau(h) = \tau_n + (\tau_r - \tau_n)(1 - e^{-(h/r)})$ (A2d)

$$\text{Spherical: } \tau(h) = \left[\begin{array}{ll} \tau_r & \text{for } h > r \\ \tau_n + (\tau_r - \tau_n)\left[\dfrac{3h}{2r} - \dfrac{h^3}{2r^3}\right] & \text{for } h \le r \end{array}\right. \qquad \text{(A2e)}$$

The estimate of a data value at an unmeasured point Y is a weighted sum of the available measurements:

$$\hat{Y} = \sum_{i=1}^{m} w_i Y_i \qquad \text{(A3)}$$

where w_i is the weight for measurement Y_i, m is the number of measurements, and

$$\sum_{i=1}^{m} w_i = 1 \qquad \text{(A4)}$$

Kriging is the algorithm for determining the weights w_i such that the estimate has minimum variance. This is a Lagrangian optimization problem, which requires the solution of a system of linear equations. This system is:

$$\Gamma\mathbf{w} = \Gamma_Y$$

where:

$$\Gamma = \left[\begin{array}{ccccccc|c} & & & \cdot & & & & 1 \\ & & & \cdot & & & & 1 \\ & & & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \tau(h_{ij}) & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & & & & \cdot \\ & & & \cdot & & & & \cdot \\ & & & \cdot & & & & 1 \\ \hline 1 & 1 & \cdot & \cdot & \cdot & \cdot & 1 & 0 \end{array}\right] \qquad \text{(A6a)}$$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ w_m \\ \lambda \end{bmatrix} \qquad \text{(A6b)}$$

$$\Gamma_Y = \begin{bmatrix} \tau(h_{Y1}) \\ \tau(h_{Y2}) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \tau(h_{Ym}) \\ 1 \end{bmatrix} \qquad \text{(A6c)}$$

Also, h_{ij} = distance between measurement sites i and j, h_{Yj} = distance between the point being estimated and measurement site j, and λ is the Lagrange parameter. The 1's in the right-hand column of the Γ matrix causes the Lagrange parameter λ to be added to each equation in the system. The row of 1's at the bottom of the Γ, matrix and the 1 at the bottom of the Γ_Y vector provides the equation that causes the sum of the weights w_i to equal unity. The solution $\mathbf{w}$ to this system gives the weights to be used on the measurements to estimate the data value at point Y.

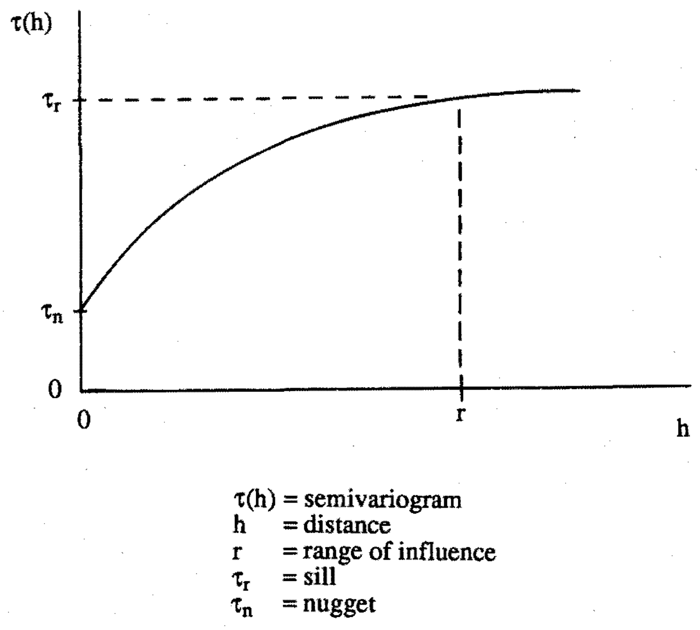

Figure A1. Typical Shape of a Semivariogram and Its Characteristic Values.

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